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# On the economic theory of crop rotations: value of the crop rotation effects and implications on acreage choice modeling

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**Abstract**

Crop rotations are known to have two main kinds of economic effects: direct effects on potential yields and on the productivity of different inputs, and indirect effects on economically optimal input levels, especially pesticides and fertilizers. The main objective of this article is to uncover the mechanisms through which crop rotation effects affect the acreage choices of forward-looking farmers, in a dynamic programming framework. Whereas most models considering acreage choices with crop rotation effects are based on discrete choice models at the plot level, our model considers a farm level strategy. This implies that our theoretical modeling framework is closely related to the models commonly used for empirically investigating farmers' acreage choices, either in the multicrop econometric literature or in the mathematical programming literature. We provide original results aimed at characterizing the properties of optimal acreage choices accounting for crop rotation effects and constraints in an uncertain context. Using a stochastic programming approach together with a Lagrangian approach we show that optimal dynamic acreage choices can be formally characterized as static acreage choices with contingent renting/lending markets for acreages with specific preceding crops. The crop rotation constraint Lagrange multipliers provide the renting/lending prices of acreages with specific crop histories. The results presented in the article are mainly theoretical. Our modeling framework can easily be implemented in practice since it mainly considers quadratic programming problems and their solution functions.

**Keywords:** crop rotation, constrained optimization, dynamic programming, stochastic programming, dynamic acreage choice model

**JEL classifications:** C61, D21, D24, D92, Q12

**Micro-économie des successions culturales : valeur économique des successions culturales et dynamique des choix d'assolement**


**Résumé**

La prise en compte des effets des précédents culturaux jouent à deux niveaux sur les choix de production. Les précédents culturaux impactent directement les rendements potentiels des cultures (courants et futurs) et la productivité (courante et future) d'intrants variables, des pesticides et des engrais en particulier. Ceci entraîne que les précédents culturaux doivent également affecter les choix d'assolement (courants et futurs) des agriculteurs : les précédents culturaux affectent les marges (courantes et futures) des cultures assolées et les surfaces en précédents culturaux sont déterminées par les choix d'assolement passés. Le principal objectif de cet article est d'analyser les mécanismes selon lesquels les effets des précédents culturaux affectent les choix d'assolement des agriculteurs. Le choix d'assolement de la campagne en cours doit tenir compte (*i*) de la disponibilité des surfaces de précédents culturaux, (*ii*) des effets de ces précédents sur les marges de la campagne en cours et (*iii*) les choix d'assolement de la campagne en cours détermineront les surfaces en précédents des campagnes à venir. Alors que la plupart des analyses des effets des précédents culturaux sur les choix de culture sont fondés sur l'utilisation de modèles de choix discret « à la parcelle », l'analyse conduite ici considère des stratégies de choix d'assolement définies à l'échelle de l'exploitation agricole. Les modèles développés ici peuvent donc être analysés comme des extensions « dynamiques » des modèles de choix d'assolement « statique » considérés habituellement par les économistes de la production agricole. Notre approche combine des éléments de programmation stochastique et de programme sous contraintes. Elle permet d'obtenir des résultats originaux caractérisant les propriétés des choix d'assolements tenant compte des effets et des contraintes des précédents culturaux en avenir incertain (en horizon fini et avec un espace d'états discret/étisé). En particulier, nous montrons que les choix d'assolement dynamiques optimaux peuvent être décrits à partir de modèles usuels de choix d'assolement statique dès lors que sont introduits des marchés contingents virtuels pour la location de surfaces de précédents culturaux. Les multiplicateurs de Lagrange des contraintes liées aux surfaces de précédents culturaux du problème d'optimisation des choix d'assolement dynamiques sont alors les prix de location de ces surfaces sur les marchés contingents virtuels. Leur niveau optimal garantit l'égalité entre l'offre et la demande de précédents culturaux dans tous les scénarios possibles. Les résultats d'analyse présentés ici sont essentiellement théoriques. Ils peuvent néanmoins être aisément appliqués puisqu'ils sont développés dans un cadre d'analyse standard. Les problèmes d'optimisation dynamique des choix d'assolement avec effets des précédents culturaux sont ici présentés sous la forme de problèmes d'optimisation quadratique. Nos résultats présentent les spécificités de ces problèmes et de leurs fonctions solutions.

**Mots-clés :** successions culturales, optimisation sous contraintes, programmation dynamique, programmation stochastique, modélisation dynamique des choix d'assolement

**Classification JEL :** C61, D21, D24, D92, Q12

**On the economic theory of crop rotations: value of the crop rotation effects and implications on acreage choice modeling**

## 1. Introduction

In their critical literature review on the theory and measurement of farmers' choices, Just and Pope (2002) forcefully and convincingly argue that "*potentially large gains may come from understanding more of the structure that underlies the production technology for investigating and modeling farmers' choices*". In particular these authors ask: "*What elements of technology should economists consider essential?*" As far as crop acreage choices are concerned, a consensus seems to exist among agricultural scientists and extension agents: crop rotation effects and the related constraints are major determinants of farmers' crop choices. Moreover the increasing concerns related to the impacts of agricultural production on the environment (as well as on public health) suggest that more efforts should be put on the investigation of the impacts crop rotation effects and constraints on farmers' choices. Crop rotation effects are the key stone of many cropping systems aimed at reducing the use of chemical inputs. The design of economic instruments aimed at stimulating the adoption of these environmentally friendly cropping practices relies on a better understanding of the impacts of the dynamic features of the agricultural production process on farmers' production choices.

Previous analyses of the impacts of crop rotation effects on farmers' acreage choices generally focus on specific case studies and as a result, consider specifically designed modeling frameworks. Our main aim is to analyze the mechanisms through which crop rotation effects and constraints affect forward-looking farmers' acreage choices. Our focusing on these mechanisms distinguishes our work from the previous ones on this topic.

To this end, we consider acreage choices within a dynamic stochastic programming framework and we present a theoretical analysis of this problem aimed at illustrating the role of crop rotation effects and constraints on farmer's acreage choices. Our models are based on quadratic programming problems for three main reasons. First, quadratic objective functions can accommodate a large scope of problems. Second, the solution functions to parametric quadratic programming problems can, to some extent, be characterized using analytical results. Third, quadratic programming problems, even very large ones, can be solved by using any optimization software.

Crop rotation effects are generally ignored in the acreage choice models found in the agricultural production economics literature. Most of the papers dealing with crop rotation effects focus on stock management issues or/and on the effects of specific crop sequences. Whereas Thomas (2003) focuses on nutrient management issues, the approaches considered in Ecktein (1984), Ozarem and Miranovski (1994) or McEwan and Howitt (2011) aggregate the crop rotation effects in a "fertility index". Such a "fertility index" is well suited for modeling the effects of crop rotation which are coproducts of a previous crop and which will be inputs for any succeeding crop. Nutrients surpluses and soil structure effects meet these requirements. Crop rotation effects related to pest management need to be handled in a different manner. Any crop affects the pest and weed populations present in a considered plot. But these impacts affect differently the succeeding crops, depending on the impacts of the considered pest and weed populations on these succeeding crops.

Due to crop rotation effects related to pest management, we prefer to consider crop rotation effects at the crop sequence level. The modeling framework developed by El-Nazer and McCarl (1986) also considers crop sequences. However, their approach implicitly assumes that farmers commit on long crop sequences. It seems more relevant to consider that farmers adapt their acreage choices continuously over time according to the evolution of the economic and bio-climatic contexts, as in Hendricks *et al.* (2014) or in Livingston *et al.* (2008, 2013).

If crop rotation effects are generally not explicitly considered in agricultural production choice models, they are often used for arguing assumptions underlying these models, either in the Multicrop Econometric literature (ME) (see, *e.g.*, Chambers and Just, 1989) or in the (Positive) Mathematical programming (P)MP (see, *e.g*., Howitt, 1995 ; Heckeleï and Wolff, 2003). These assumptions often motivate crop diversification. Crop rotation effects partly underlie the crop acreage bounds used in MP models, the implicit cost function used in the (P)MP literature or the decreasing marginal return to crop acreage assumption used in both the PMP and ME literatures.[1]

Oude Lansink and Stefanou (2001) consider dynamic acreage choice models but from a fairly different viewpoint. They consider exogenous acreage adjustment costs but ignore the effects of crop rotation *per se*. To our knowledge, Hennessy (2006) is the only author addressing the issues

[1] Of course agricultural scientists extensively deal with crop rotation effects. If their analyses focus on biological aspects, a growing literature addresses acreage optimization problems involving crop rotation effects and constraints as shown by the recent survey of Dury *et al*. (2012). These studies generally propose a modeling framework followed by an empirical application on a case study (see, *e.g*., Cai *et al*., 2013 ; Schönhart *et al*., 2011 ; and Detlefsen and Jensen, 2007). The modeling framework proposed by Salassi *et al* (2013) is the most closely related to ours. But these studies do not seek to provide a general analysis of the mechanisms through which crop rotation affects the optimal acreage choices. To provide such a general analysis is the main objective of this article.

raised by the identification and the optimal use of crop rotation effects lasting for more than one year.[2]

Accounting for crop rotations in acreage choice models is especially challenging. First, a dynamic programming framework must be employed because dynamic crop acreage choices involve inter-temporal trade-offs. Second, crop rotations involve many constraints in addition to the usual acreage non-negativity constraints. Third, crop rotation effects in the crop production technologies must be represented in addition to the other determinants of acreage choices, *e.g*. management of labor and capital at peak-times or economic risk spreading.

Our considering acreage optimization strategies at the farm level distinguishes our work from that of McEwan and Howitt (2011), Hendricks *et al*. (2014) and Livingston *et al*. (2008, 2013). In these recent studies acreage choices are defined as sums of discrete choices defined at the plot level. By addressing farm level strategy we will provide further insights on the practice of acreage choice modeling. Hennessy (2004) also considered the effects of crop rotations on acreage choices according to a farm level strategy involving issues. But his analysis was essentially static. It was focused on the optimal use of crop rotation effects on one hand, and on farm income risk management and labor management on the other hand.

Our analysis is based on a set of mathematical results characterizing optimal dynamic acreage choices as the solutions to a particular dynamic programming problem. The purpose of this article is not to highlight the proofs of these results but rather to develop their economic interpretations.

[2] See also Klein Haneveld and Stegeman (2005) for a related study from an operation research viewpoint.

The related proofs are detailed in the Technical Appendix which is available from the authors upon request[3]. These results heavily rely on results published in the operations research literature. In particular, we extensively use the results of Bemporad *et al* (2002) and of Berkelaar *et al* (1997). These authors consider sensitivity analysis of quadratic programming problems for characterizing the solutions to the so-called "multiparametric quadratic programming problems". We use these comparative statics results for characterizing the solutions to dynamic programming problems involving quadratic per period objective functions and linear constraints. Our results are also closely related to those obtained by Rockafellar (1999) for general multistage stochastic programming problems casted in the so-called "extended linear-quadratic framework".

The first section considers the modeling of the crop rotation effects at the crop sequence level and uncovers the determinants of the economic value of the crop rotation effects. The second section defines the problem considered by a farmer when choosing his acreage while accounting for crop rotation effects and constraints. The next sections aim at characterizing the solutions to this dynamic programming problem denoted as problem (D). We proceed in three steps. The third section considers a simple static acreage choice problem and its solution functions. This problem is denoted as problem (S). Its solution functions are then used for providing simple characterizations of the solutions to more complicated problems. In the fourth section we analyze the myopic acreage choice problem, denoted as problem (M). Myopic acreage choices account for crop rotation constraints, as it is the case for dynamic acreage choices, but they ignore the effects of the current acreage choices on the future profit stream, as it is the case for static acreage

[3] The supplementary material is available on the website http://www.rennes.inra.fr/smart_eng/Working-Papers-SMART-LERECO

choices. The analysis of the myopic problem is an intermediate step from the simple static problem (S) to the dynamic programming problem (D). The fifth section is devoted to the analysis of problem (D) and to the characterization of its solutions. We proceed by employing a Lagrangian approach for handling the crop rotation constraints in problems (M) and (D). The Lagrange multipliers of these constraints are particularly useful for characterizing the solutions to these problems (M) and (D) when combined with the simple solution functions to problem (S). We also stress out that these Lagrange multipliers have particularly interesting economic, *e.g.* market based, interpretations as implicit prices of the crop rotation effects. Some extensions of our modeling framework and some implications of our results are discussed in the sixth section.

## 2. Crop rotations effects: yield levels, input uses and gross margins

The preceding crop of a crop grown on a given plot can generate three main types of effects. It partially determines (*i*) the level of pest and disease, as well as weed, infestation levels, (*ii*) the nutrient levels available at the beginning of the cropping process and (*iii*) the soil structure of the plot.[4] These effects can directly affect the yield levels as well as the input productivity levels. To explicitly model these effects would be a considerable task. But these effects can be incorporated in an agricultural production technology model in an approximate framework, by considering crop sequences.

We assume for the ease of exposition that the agricultural production process is dynamic of order 1. Let $\tilde{y}_{mk,t}(\mathbf{x}_k)$ denote the yield at date $t$ of crop $k$ on plots with previous crop $m$ when variable

[4] And, as a consequence, the soil properties with respect to the development of the plant root system, with respect to the soil flora and fauna or with respect to water drainage or holding capacity.

inputs are used in the quantities given in $\mathbf{x}_k$. This yield level is random because it depends on random cropping conditions. First, this yield depends on the cropping conditions induced by the previous crop. These effects are denoted by the random vector $\tilde{\mathbf{c}}_{m,t}$ if crop *k* is planted on a plot with previous crop *m*. The term $\tilde{\mathbf{c}}_{m,t}$ contains, *e.g.*, the nutrient surpluses left by crop *m*, the effects of previous crop *m* on the pest and weed populations affecting the yield of crop *k*, *etc*. Second, the yield of crop *k* also depends on the effects exogenous random events such as, *e.g.*, climatic conditions. Denoting these random events by $\tilde{\mathbf{e}}_t$, $\tilde{y}_{mk,t}(\mathbf{x}_k)$ can be defined as:

$$\tilde{y}_{mk,t}(\mathbf{x}_k) = f_k(\mathbf{x}_k; \tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_t) \qquad (1)$$

where $f_k(.;.)$ is a "generalized" yield function of crop *k*. The effects of $\tilde{\mathbf{c}}_{m,t}$ on $\tilde{y}_{mk,t}(\mathbf{x}_k)$ define the crop rotation effects of previous crop *m* on the yield of crop *k* at date *t*. They differ from that of previous crop *n* if $\tilde{\mathbf{c}}_{n,t} \neq \tilde{\mathbf{c}}_{m,t}$.

In order to investigate the economic impacts of crop rotation we need to specify farmers' behavior and information set. Let $\tilde{p}_{k,t}$ denote the price of crop *k* at date *t*, and $\tilde{\mathbf{w}}_{k,t}$ denote that of the variable input price vector. Let $\upsilon_{mk,t}$ define the information available to the farmer when he chooses his variable input levels $\mathbf{x}_k$ at date *t*. Of course $\upsilon_t$ includes the realization of $\tilde{\mathbf{w}}_{k,t}$. It also contains the realizations of elements of $(\tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_t)$ and, more generally, informative signals related to the realization of $(\tilde{p}_{k,t}, \tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_t)$. $\upsilon_{mk,t}$ is a realization of the random variable $\tilde{\upsilon}_{mk,t}$. Assuming that the considered farmer is risk neutral, his optimal input choice maximizes the expectation conditional on $\upsilon_{mk,t}$ of the gross margin of crop *k* with previous crop *m*, *i.e.* the optimal input choice is determined by:

$$\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t}) \equiv \arg\max_{\mathbf{x}_k \geq \mathbf{0}} E_{\upsilon_{mk,t}} \tilde{\pi}_{mk,t}(\mathbf{x}_k; \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t}) \tag{2}$$

where $\tilde{\pi}_{mk,t}(\mathbf{x}_k; \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t}) \equiv \tilde{p}_{k,t}\tilde{y}_{mk,t}(\mathbf{x}_k) - \tilde{\mathbf{w}}'_{k,t}\mathbf{x}_k$. The term $E_{\upsilon_{mk,t}}$ denotes the expectation operator conditional on $\upsilon_{mk,t}$.

The gross margin obtained by the farmer is then the random variable defined by:

$$\tilde{\pi}^{*}_{mk,t}(\upsilon_{mk,t}) \equiv \tilde{\pi}_{mk,t}[\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t}); \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t}] = \tilde{p}_{k,t}\tilde{y}^{*}_{mk,t}(\upsilon_{mk,t}) - \tilde{\mathbf{w}}'_{k,t}\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t}) \tag{3}$$

while the obtained yield is defined by $\tilde{y}^{*}_{mk,t}(\upsilon_{mk,t}) \equiv f_k[\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t}); \tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_t]$. The gross margin of crop *k* with previous crop *m* depends on the crop rotation effects of crop *k* on previous crop *m* through two main (interrelated) channels. First, $\tilde{\pi}^{*}_{mk,t}(\upsilon_{mk,t})$ depends on $\tilde{\mathbf{c}}_{m,t}$ through its (direct) effects on the yield function of crop *k* at $\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t})$, *i.e.* on $\tilde{y}^{*}_{mk,t}(\upsilon_{mk,t})$. These effects include, *e.g.*, the crop rotation effects related to pest infestations against which no commercial pesticide is available. Second, $\tilde{\pi}^{*}_{mk,t}(\upsilon_{mk,t})$ depends on $\tilde{\mathbf{c}}_{m,t}$ through its effects on the optimal variable input choices $\mathbf{x}^{*}_{mk,t}(\upsilon_{mk,t})$. The considered farmer chooses $\mathbf{x}_k$ according to his knowledge on the interactions of $\tilde{\mathbf{c}}_{m,t}$ and $\mathbf{x}_k$ in $\tilde{y}^{*}_{mk,t}(\upsilon_{mk,t})$. This results in higher productivity levels of $\mathbf{x}_k$. Depending on the effects of $(\tilde{\mathbf{c}}_{m,t}, \mathbf{x}_k)$ in $\tilde{y}^{*}_{mk,t}(\upsilon_{mk,t})$, on input savings or additional expenses. If crop *m* leaves large nutrient amount to the succeeding crops, then to account for crop rotation effects with previous crop *m* results in fertilizer savings. If crop *m* reduces (increases) the levels of the pest population affecting crop *k*, then to account for crop rotation effects of crop *k* with previous crop *m* results in pesticide savings (additional expenses).

The differences in crop sequence gross margins $\tilde{\pi}^{*}_{mk,t}(\upsilon_{mk,t})$ across crop sequences provide the basis for the usual crop rotation schemes designed by agricultural scientists and extension agents. From an economic viewpoint these rotation schemes can be interpreted as rules of thumb which are approximately optimal in given price ranges. Of course high crop prices tend to magnify the economic value of the direct effects of crop rotation on yields, while high input prices magnify those on input uses. These price effects need to be considered when designing economic policies aimed at promoting environmentally friendly agricultural production practices. *E.g.*, pesticide taxes clearly provide incentives for taking advantage of crop rotation effects which are pesticide use reducing. But high output prices tend to reduce the incentives for reducing input uses through crop rotations.

## 3. Dynamic acreage choices: dynamic programming and crop rotation constraints

This section proposes a modeling framework for investigating optimal acreage choices accounting for crop rotation effects and constraints. It is fairly general and, as will be seen below, can be further extended. However, it relies on several specific assumptions with respect to the production technology which are discussed below.

As in the preceding section we assume production dynamics of order 1 even if the proposed framework can easily be extended to dynamics of higher order, at least in theory. The considered crop bundle is assumed to be fixed. It is denoted by $\mathcal{K} \equiv \{1,...,K\}$. The expected return of crop $k$ (for $k \in \mathcal{K}$) on plots with previous crop $m$ (for $m \in \mathcal{K}$) is described by the random variable $\tilde{\pi}_{mk,t}$. $\tilde{\pi}_{mk,t}$ is to be interpreted as the random gross margin obtained by a farmer optimizing his variable input choices at the crop level subject to the technological constraint described in equation (1). If the farmer is considered as risk neutral as assumed in the previous section, $\tilde{\pi}_{mk,t}$ is defined by:

$$\tilde{\pi}_{mk,t} \equiv \tilde{\pi}_{mk,t}[\mathbf{x}^{*}_{mk,t}(\tilde{\upsilon}_{mk,t}); \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t}] \qquad (4)$$

where $\tilde{\pi}_{mk,t}(.;.)$ is defined in equation (3).

The probability distribution of the random vector $(\tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_{t}, \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t}, \tilde{\upsilon}_{mk,t})$ determines that of $\tilde{\pi}_{mk,t}$. This probability distribution describes how the considered farmer perceives at date 1 the random elements which will affect his yields and input choices at date t. In particular, the farmer knows that some information set will be available to him at date *t*. But this information set is unknown at date 1. At date 1 the farmer just knows that he will receive an information set according to the probability distribution defined by that of $\tilde{\upsilon}_{mk,t}$ and that we will choose in variable input levels for crop *k* on previous crop *m* according to the probability distribution of $(\tilde{\mathbf{c}}_{m,t}, \tilde{\mathbf{e}}_{t}, \tilde{p}_{k,t}, \tilde{\mathbf{w}}_{k,t})$ conditional on the realization of $\tilde{\upsilon}_{mk,t}$.

The $\tilde{\pi}_{mk,t}$ terms are stacked in the $\tilde{\boldsymbol{\pi}}_{k,t} \equiv (\tilde{\pi}_{mk,t} : m \in \mathcal{K})$ vectors for $k \in \mathcal{K}$ which are themselves stacked in the $\tilde{\boldsymbol{\pi}}_{t} \equiv (\tilde{\boldsymbol{\pi}}_{k,t} : k \in \mathcal{K})$ vectors. The differences in the elements of $\tilde{\boldsymbol{\pi}}_{k,t}$ terms describe the crop rotation effects related to the preceding crops of crop *k*.

In order to account for these effects farmers need to consider acreage choices at the crop sequence level. Let $s_{mk,t}$ be the acreage of crop *k* planted in year *t* on plots with preceding crop *m*. Let $\mathbf{s}_{k,t} \equiv (s_{mk,t} : m \in \mathcal{K})$ denote the acreage vector describing the use of preceding crop acreage for crop *k* in year *t*, and let $\boldsymbol{\iota}$ denote the *K* dimensional unitary vector. The term $a_{k,t} \equiv \boldsymbol{\iota}'\mathbf{s}_{k,t}$

defines the (total) acreage of crop *k* in year *t*. The crop rotation constraints in year *t* state that the use of preceding crop acreages must equal their corresponding supplies, *i.e.* $\boldsymbol{\iota}'\mathbf{s}_{(m),t} = a_{m,t-1}$ must hold for $m \in \mathcal{K}$ where $\mathbf{s}_{(m),t} \equiv (s_{mk,t} : k \in \mathcal{K})$. The matrices $\mathbf{A} \equiv \mathbf{I} \otimes \boldsymbol{\iota}'$ and $\mathbf{D} \equiv \boldsymbol{\iota}' \otimes \mathbf{I}$, where $\mathbf{I}$ denotes the *K* dimensional identity matrix, allow defining the crop rotation constraints in year *t* in the following compact form:

$$\mathbf{D}\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} = \mathbf{a}_{t-1} \tag{5}$$

where $\mathbf{s}_t \equiv (\mathbf{s}_{k,t} : k \in \mathcal{K})$ and $\mathbf{a}_t \equiv (a_{k,t} : k \in \mathcal{K})$. Note that $\mathbf{D}\mathbf{s}_t$ can be defined as $\mathbf{d}_t \equiv \mathbf{D}\mathbf{s}_t$ with $\mathbf{d}_t \equiv (d_{m,t} : m \in \mathcal{K})$ and $d_{m,t} = \boldsymbol{\iota}'\mathbf{s}_{(m),t}$ being interpreted as the "demand" of acreage with preceding crop *m* induced by $\mathbf{s}_t$.

As stated in our description of the agricultural process in the previous section, it is assumed that the production technology of any crop sequence $(m,k)$ exhibits constant return to acreage. *I.e.*, the $f_k(\mathbf{x}_k;\tilde{\mathbf{c}}_{m,t},\tilde{\mathbf{e}}_t)$ functions do not depend on $\mathbf{s}_t$. Although constant returns to crop acreages is a maintained assumption in models considering risk spreading as a diversification motive (see, *e.g.*, Hazell and Norton, 1986 ; Chavas and Holt, 1990 and 1996 ; Holt, 1999), marginal returns to crop acreages are often assumed decreasing in the acreage of the considered crop (see, *e.g.*, Just *et al*, 1983 ; Howitt, 1995). This assumption is often based on crop rotation arguments: the larger the acreage of a given crop is, the less this acreage can exploit crop rotation effects. These crop rotation considerations are explicitly accounted for in the present framework as it is related to the "effect" of $a_{k,t} = \mathbf{s}'_{k,t}\boldsymbol{\iota}$ on $\mathbf{s}'_{k,t}\tilde{\boldsymbol{\pi}}_{k,t}$ assuming that $\mathbf{s}_{k,t}$ is optimally chosen.

According to this assumption farmers' acreage choices do not affect their variable input choices at the crop sequence level.[5] This implies that the crop variable input level choices and the acreage choices can be separately analyzed. This section and the remaining ones focus on the later choices.

Finally we consider that farmers adopt a farm level strategy when deciding their acreage with a per period objective function of the form:

$$\Pi(\mathbf{s}_t;\tilde{\boldsymbol{\pi}}_t) \equiv \mathbf{s}_t'\tilde{\boldsymbol{\pi}}_t - C(\mathbf{s}_t)\,. \tag{6}$$

$C$ is assumed to be convex and strictly increasing in $\mathbf{s}_t \geq \mathbf{0}$. Due to its convexity in $\mathbf{s}_t$ the term $C(\mathbf{s}_t)$ provides incentives for diversifying the acreages. By contrast the weighted sum of the gross margins $\mathbf{s}_t'\tilde{\boldsymbol{\pi}}_t$ provides incentives for allocating the preceding acreage to the most profitable crop sequences.

In the present framework we define the term $C(\mathbf{s}_t)$ as an implicit management cost function which can be interpreted as a smooth reduced form approximation of the shadow costs generated by the quasi-fixed factor limiting quantities. Quasi-fixed factor constraints generate implicit costs depending on farmers' acreage choices because a large acreage of a given crop generates costly peak loads for labor and machinery. Cost functions similar to $C(\mathbf{s}_t)$ are used in the multicrop econometric models considered by Carpentier and Letort (2012, 2014) and is sometimes defined

[5] As discussed in Carpentier and Letort (2012, 2014) this assumption is reasonable in cases where farmers are more reluctant to change their cropping practices, at the crop sequence level, than to change their acreages. This is a stylized fact often reported by agricultural scientists and extension agents.

as the "PMP term" in the PMP literature (Heckeleï *et al*, 2012). The term $\Pi(\mathbf{s}_t;\tilde{\boldsymbol{\pi}}_t)$ will be referred to as a profit function. Extensions of this simple periodic objective function are discussed later.

To optimally choose acreages accounting for crop rotation effects and constraints consists in solving the following dynamic programming problem:

$$\text{(D)} \quad \max_{\tilde{\mathbf{s}}\geq \mathbf{0}} \left\{ E_{\omega_1} \sum\nolimits_{t=1}^{T} \beta^{t-1} E_{\tilde{\omega}_t} \Pi(\tilde{\mathbf{s}}_t;\tilde{\boldsymbol{\pi}}_t) \text{ s.t. } \mathbf{D}\tilde{\mathbf{s}}_t = \mathbf{A}\tilde{\mathbf{s}}_{t-1} \text{ for } t=1,...,T \right\}$$

with $\tilde{\mathbf{s}} \equiv (\tilde{\mathbf{s}}_t : t=1,...,T)$ and with the convention $\mathbf{A}\tilde{\mathbf{s}}_0 \equiv \mathbf{a}_0$. The objective function of the considered problem is the expected discounted sum of profits where $\beta \in (0,1)$ is the discount factor and $T$ is the considered horizon which is assumed to be finite. The previous crop acreage at date $t=1$, $\mathbf{a}_0 \in \mathcal{A}$, is a parameter of the considered problem.

The land allocation vector $\tilde{\mathbf{s}}$ defines the set of control variables of problem (D). The state of the process to be optimized is described by two state variables at date $t$: the exogenous state variable $\tilde{\omega}_t$ and the endogenous state variables $\tilde{\mathbf{a}}_{t-1} = \mathbf{A}\tilde{\mathbf{s}}_{t-1}$. The state variable $\tilde{\mathbf{a}}_{t-1}$ is endogenous because it is determined by the control variable $\tilde{\mathbf{s}}_{t-1}$. With $\boldsymbol{\iota}'\mathbf{a}_0 = S > 0$ any feasible solution to problem (D) satisfies $\mathbf{j}'\tilde{\mathbf{s}}_t = S$ for $t=1,...,T$, where $\mathbf{j} \equiv \boldsymbol{\iota} \otimes \boldsymbol{\iota}$ is the dimension $N$ unitary vector. The exogenous state $\tilde{\omega}_t$ is defined as a random variable with support $\mathcal{W}_t$. The realizations $\omega_t \in \mathcal{W}_t$ of the random terms $\tilde{\omega}_t$ are defined as the information sets containing the information relevant for updating the probability distribution of the current and future gross margins $\tilde{\boldsymbol{\pi}}_\tau$ for $\tau = 1,...,t$.[6] Information sets are rarely explicitly introduced in the description of dynamic programming problems. They are considered here because the terms $\omega_t$ have a convenient interpretation as scenarios in the finite horizon framework with discrete exogenous state variables, the case considered later.

The probability distribution of the random terms $\tilde{\boldsymbol{\pi}}_t$, $\tilde{\mathbf{s}}_t$ and $\tilde{\omega}_t$ terms in the definition of problem (D) is conditional on the fixed initial information set $\omega_1$. *I.e.*, $\tilde{\omega}_t$ describes the probability distribution of the information set which will be available at date *t* as anticipated at date 1. The acreage choices $\tilde{\mathbf{s}}_t$ are defined as random variables because their optimal choices, denoted by $\tilde{\mathbf{s}}_t^o$, are to be defined as optimal responses to the exogenous random state variables $\tilde{\omega}_t$. The only exception is the acreage choice at date 1, $\mathbf{s}_1$, which is fixed because $\omega_1$ is known at date 1, *i.e.* $\mathcal{W}_1 \equiv \{\omega_1\}$.

The term $E_{\tilde{\omega}_t}$ denotes the expectation operator conditional on the information set $\tilde{\omega}_t$. The formulation of problem (D) indicates that the farmer knows that he will get information signals each year and will update his information set accordingly, and then make use of this information

[6] The dynamic programming problems found in the literature usually relies on assumptions related to the stochastic process generating the exogenous state variables of the problem, *e.g.* Markov and stationary assumptions. Such assumptions are useful, if not necessary, for simplifying the analysis of long horizon problems but they are less crucial for short horizon problems. Moreover, various public interventions almost continuously affect agricultural markets and the economic incentives of farmers. In short horizon problems the impact of these interventions act as exogenous random shocks, *e.g.* on the prices of the agricultural commodities, which are difficult to model and which can affect large parts of the considered horizon.

set for optimizing his acreage choices. The fact that the realizations of the control variables $\tilde{\mathbf{s}}_{\tau}$ for $\tau = 1,...,t-1$ are known is implicit. The expectation of $\tilde{\boldsymbol{\pi}}_t$ conditional on the information set $\tilde{\omega}_t$ received at date *t*, denoted as $\bar{\boldsymbol{\pi}}_{t|\tilde{\omega}_t} \equiv E_{\tilde{\omega}_t}\tilde{\boldsymbol{\pi}}_t$, is an exogenous state variable which is part of $\tilde{\omega}_t$.[7] Thanks to the linearity of $\Pi(\mathbf{s}_t;\tilde{\boldsymbol{\pi}}_t)$ in $\tilde{\boldsymbol{\pi}}_t$ we have $E_{\tilde{\omega}_t}\Pi(\mathbf{s}_t;\tilde{\boldsymbol{\pi}}_t) = \Pi(\mathbf{s}_t;\bar{\boldsymbol{\pi}}_{t|\tilde{\omega}_t})$.

The next sections aim at characterizing the solutions to problem (D) under the assumptions that the cost function $C$ is quadratic in $\mathbf{s}_t$ and that the distribution of the $\tilde{\omega}_t$ terms is discrete with finite support. In particular, we aim at showing that the solutions to problem (D) can be characterized by using the solutions functions of a standard static acreage choice problem and that this characterization provides simple and useful economic interpretations.

## 4. Static acreage choices

We first characterize the solutions to the following static problem:

(S) $\max_{\mathbf{s}\geq\mathbf{0}} \Pi(\mathbf{s};\boldsymbol{\pi})$ where $\mathbf{s} \in \mathbb{R}_+^N$ and $\boldsymbol{\pi} \in \mathbb{R}^N$.

Considering crop sequences apart this problem is fairly standard. It ignores the dynamic aspects of farmers' acreage choice, *i.e*. the crop rotation effects and constraints. The only feature of problem (S) which is not usual is that the choice of $\mathbf{s}$ is not constrained by a total land use constraint. A total land use constraint would not change the main results presented in this section.

[7] Although it partially depends on the variable input choices of the farmer, the expected gross margin vector $\bar{\boldsymbol{\pi}}_{t|\omega_t}$ is exogenous in problem (D) because it is not controlled by the farmer through his acreage choices.

But to ignore this constraint allow simpler characterizations of the solutions to the more complicated myopic and dynamic problems. As noted above, to account for crop rotation constraints automatically enforces a total land use constraint. As a result, problems (M) and (D) are implicitly constrained by the available land of the considered farm.

The cost function $C$ is assumed to be quadratic in $\mathbf{s}$. This functional form is usual as it can be considered as a smooth second order (local) approximation to any sufficiently smooth cost function. Quadratic cost functions are employed in the PMP literature (see, *e.g.*, Heckeleï *et al*, 2012) and in the multicrop econometric literature (see, *e.g.*, Carpentier and Letort, 2012). Quadratic functions such as $\Pi(\mathbf{s};\boldsymbol{\pi}) \equiv \mathbf{s}'\boldsymbol{\pi} - C(\mathbf{s})$ are also convenient as period objective functions in dynamic programming problems, as shown by the extensive use of quadratic cost functions in the partial adjustment literature (see, *e.g.*, Epstein 1981 ; Oude Lansink and Stefanou 2001) or in the operations research literature (see, *e.g.*, Bemporad *et al* 2002 or Rockafellar 1999). In particular, $C$ being quadratic in $\mathbf{s}$ problem (S) is a quadratic programming problem, *i.e.* a workhorse of the optimization literatures. The theoretical properties of quadratic programming are fairly well documented and any optimization software contains efficient quadratic programming algorithms.

The cost function C is assumed to be quadratic and strictly convex in $\mathbf{s}$, *i.e.*:

$$C(\mathbf{s}) = \mathbf{h}'\mathbf{s} + 1/2 \times \mathbf{s}'\mathbf{H}\mathbf{s} \text{ where } \mathbf{H} \in \mathbb{R}^N \times \mathbb{R}^N \text{ is positive definite, } \mathbf{h} \in \mathbb{R}^N \text{ and } \mathbf{s} \in \mathbb{R}_+^N. \quad (7)$$

$C$ can also be chosen strictly increasing in $\mathbf{s}$ on $\mathbb{R}_+^N$. This can be guaranteed by assuming, *e.g.*, that $\mathbf{h}$ and $\mathbf{H}$ are both positive. The strict convexity assumption mainly aims at avoiding multiple solutions in $\mathbf{s}$ to the considered optimization problems. It significantly simplifies the analysis as it allows using standard differential calculus (see, *e.g.*, Fiacco and Kyparisis, 1985).

Given our interpretation of $C(\mathbf{s})$ the assumption stating its strict convexity in $\mathbf{s}$ is debatable. Cropping practices vary much more across crops than they vary across preceding crops for a given crop. Hence, it seems more sensible to define the acreage management cost as a function of the crop acreage vector $\mathbf{a} \equiv \mathbf{As}$, *i.e.* as $C^a(\mathbf{a}) \equiv \mathbf{h}'\mathbf{a} + 1/2 \times \mathbf{a}'\mathbf{Ga}$ where $\mathbf{G} \in \mathbb{R}^K \times \mathbb{R}^K$ is positive definite, $\mathbf{g} \in \mathbb{R}^K$ and $\mathbf{a} \in \mathbb{R}_+^K$. Of course $C^a(\mathbf{As})$ is strictly convex in $\mathbf{As}$ and convex in $\mathbf{s}$, but it is not strictly convex in $\mathbf{s}$.

Our approach consists in defining $C(\mathbf{s})$ as a strictly convex perturbed version of the "only" convex $C^a(\mathbf{As})$. The perturbation device is often used in operations research for obtaining a well-behaved objective function smoothly approximating the objective function of interest. Linear or quadratic programming problems are usually perturbed by quadratic terms. We also use a quadratic perturbation term. The solutions to the perturbed version of our problem of interest converge (in a perturbation parameter) to solutions to the problem of interest, *i.e*. the perturbed solutions are $\varepsilon-$solutions to the problem of interest. Our viewpoint is that to consider $\varepsilon-$ solutions instead of "true" solutions is a reasonable price to pay for significantly simplifying the analysis of the considered dynamic programming problems.

Proposition 1 collects a set of results characterizing the solutions to problem (S). These results are useful for analyzing the myopic and dynamic problems. We assume in this section and in the next one that the vectors $\mathbf{s}$ and $\boldsymbol{\pi}$ are structured as the vectors $\mathbf{s}_t$ and $\bar{\boldsymbol{\pi}}_{t|\omega_t}$.

**Proposition 1**. *Standard* (*or static*) *problem*. Let consider problem (S): $\max_{\mathbf{s}\geq\mathbf{0}}\Pi(\mathbf{s};\boldsymbol{\pi})$ where $\Pi(\mathbf{s};\boldsymbol{\pi})\equiv\mathbf{s}'\boldsymbol{\pi}-C(\mathbf{s})$ and $(\mathbf{s},\boldsymbol{\pi})\in\mathbb{R}_{+}^{N}\times\mathbb{R}^{N}$. Let assume that $C:\mathbb{R}_{+}^{N}\rightarrow\mathbb{R}$ is quadratic and strictly convex in $\mathbf{s}$ on $\mathbb{R}_{+}^{N}$.

(*i*) The solution in $\mathbf{s}$ to problem (S) is unique and defines the function $\mathbf{s}^{s}:\mathbb{R}^{N}\rightarrow\mathbb{R}_{+}^{N}$ by $\mathbf{s}^{s}(\boldsymbol{\pi})\equiv\arg\max_{\mathbf{s}\geq\mathbf{0}}\Pi(\mathbf{s};\boldsymbol{\pi})$. $\mathbf{s}^{s}$ is continuous in $\boldsymbol{\pi}$ on $\mathbb{R}^{N}$.

(*ii*) The solution to problem (S) defines the value function $\Pi^{s}:\mathbb{R}^{N}\rightarrow\mathbb{R}$ by $\Pi^{s}(\boldsymbol{\pi})\equiv\max_{\mathbf{s}\geq\mathbf{0}}\Pi(\mathbf{s};\boldsymbol{\pi})$. $\Pi^{s}$ is convex in $\boldsymbol{\pi}$

(*iii*) $\mathbf{s}^{s}$ is piecewise affine and $\Pi^{s}$ is piecewise quadratic in $\boldsymbol{\pi}$ on $\mathbb{R}^{N}$.

(*iv*) $\Pi^{s}$ is continuously differentiable on $\mathbb{R}^{N}$ with $\frac{\partial}{\partial\boldsymbol{\pi}}\Pi^{s}(\boldsymbol{\pi})=\mathbf{s}^{s}(\boldsymbol{\pi})$.

(*v*) $s_{mk}^{s}$ is strictly increasing in $\pi_{mk}$ at $\boldsymbol{\pi}$ if $s_{mk}^{s}(\boldsymbol{\pi})>0$. If $s_{mk}^{s}$ is constant in $\pi_{mk}$ at $\boldsymbol{\pi}$ then $s_{mk}^{s}(\boldsymbol{\pi})=0$.

Let define the functions $d_{m}^{s}:\mathbb{R}^{N}\rightarrow\mathbb{R}_{+}$ by $d_{m}^{s}\equiv\boldsymbol{\iota}'\mathbf{s}_{(m)}^{s}$ for $m\in\mathcal{K}$ and $a_{k}^{s}:\mathbb{R}^{N}\rightarrow\mathbb{R}_{+}$ by $a_{k}^{s}\equiv\boldsymbol{\iota}'\mathbf{s}_{k}^{s}$ for $k\in\mathcal{K}$. Let $\boldsymbol{\mu}\in\mathbb{R}^{K}$.

(*vi*) Let $k\in\mathcal{K}$. $a_{k}^{s}$ is strictly increasing in $\mu_{k}$ at $\boldsymbol{\pi}+\boldsymbol{\mu}\otimes\boldsymbol{\iota}$ if $a_{k}^{s}(\boldsymbol{\pi}+\boldsymbol{\mu}\otimes\boldsymbol{\iota})>0$. If $a_{k}^{s}$ is constant in $\mu_{k}$ at $\boldsymbol{\pi}+\boldsymbol{\mu}\otimes\boldsymbol{\iota}$ then $a_{k}^{s}(\boldsymbol{\pi}+\boldsymbol{\mu}\otimes\boldsymbol{\iota})=0$.

(*vii*) Let $m\in\mathcal{K}$. $d_{m}^{s}$ is strictly decreasing in $\mu_{m}$ at $\boldsymbol{\pi}-\boldsymbol{\iota}\otimes\boldsymbol{\mu}$ if $d_{m}^{s}(\boldsymbol{\pi}-\boldsymbol{\iota}\otimes\boldsymbol{\mu})>0$. If $d_{m}^{s}$ is constant in $\mu_{m}$ at $\boldsymbol{\pi}-\boldsymbol{\iota}\otimes\boldsymbol{\mu}$ then $d_{m}^{s}(\boldsymbol{\pi}-\boldsymbol{\iota}\otimes\boldsymbol{\mu})=0$.

**Proof**. See Technical Appendix.

Results (*i*)-(*iii*) are either well known or demonstrated in Bemporad *et al* (2002).[8] The strict concavity of $\Pi$ in $\mathbf{s}_t$ and the linearity of the constraints imposed in problem (S) imply that its solution in **s**, $\mathbf{s}^s(\boldsymbol{\pi})$, is unique. Result (*iii*) is a consequence of $\Pi$ being quadratic in **s** and of problem (S) involving linear inequality constraints only. It indicates that there exists a polyhedral partition $\{\mathcal{P}_j : j \in \mathcal{J}\}$ of $\mathbb{R}^N$ such that there exists $(\mathbf{b}_j, \mathbf{B}_j) \in \mathbb{R}^M \times \mathbb{R}^{M\times N}$ satisfying:

$$\mathbf{s}^s(\boldsymbol{\pi}) = \mathbf{s}^s_j(\boldsymbol{\pi}) \equiv \mathbf{b}_j + \mathbf{B}_j\boldsymbol{\pi} \text{ and } \Pi^s(\boldsymbol{\pi}) = \Pi^s_j(\boldsymbol{\pi}) \equiv \mathbf{s}^s_j(\boldsymbol{\pi})'\boldsymbol{\pi} - \mathbf{s}^s_j(\boldsymbol{\pi})'\mathbf{h} - 1/2 \times \mathbf{s}^s_j(\boldsymbol{\pi})'\mathbf{H}\mathbf{s}^s_j(\boldsymbol{\pi}) \quad (8)$$

if $\boldsymbol{\pi} \in \mathcal{P}_j$ for any $j \in \mathcal{J}$. The interior of each polyhedron *j* is characterized by a subset of crop sequences with strictly positive acreage. These crop sequence subsets define the so-called regimes of $\mathbf{s}^s(\boldsymbol{\pi})$. The full characterization of the functions $\mathbf{s}^s$ and $\Pi^s$ is provided in Proposition A1 in the Appendix.

The fact that $\Pi^s$ is continuously differentiable in $\boldsymbol{\pi}$ on $\mathbb{R}^N$, and not just "almost everywhere", might be a less standard result. Jittorntrum (1984) provides sufficient conditions ensuring the differentiability of $\Pi^s$ even at points where the non-negativity constraints imposed on the elements of **s** are just binding, *i.e.* at the regime switching points of $\Pi^s$ and $\mathbf{s}^s$.[9] These

[8] See also Lau and Womersley (2001) for similar results in a more general framework.

[9] $\Pi^s$ is twice continuously differentiable almost everywhere in $\boldsymbol{\pi}$ on P. $\Pi^s$ is twice continuously differentiable in $\boldsymbol{\pi}$ on int$\mathcal{P}_j$ for any $j \in \mathcal{J}$. $\Pi^s$ is continuously differentiable $\boldsymbol{\pi}$ on P but it is not twice differentiable in $\boldsymbol{\pi}$ on the set

conditions, *i.e*. the strict concavity of $\Pi$ in **s** and non redundant constraints on **s**, are met here.[10] The differentiability of $\Pi^s$ yields the important equation:

$$\tfrac{\partial}{\partial \boldsymbol{\pi}}\Pi^s(\boldsymbol{\pi}) = \mathbf{s}^s(\boldsymbol{\pi}) \text{ for any } \boldsymbol{\pi} \in \mathbb{R}^N \tag{9}$$

which can be interpreted as a special case of the so-called Hotelling Lemma.

Results (*v*)-(*vii*) provide the monotonicity properties of $\mathbf{s}^s$ which appear useful later by observing that $\boldsymbol{\mu} \otimes \boldsymbol{\iota} = \mathbf{A}'\boldsymbol{\mu}$ and $\boldsymbol{\iota} \otimes \boldsymbol{\mu} = \mathbf{D}'\boldsymbol{\mu}$. In particular, result (*vii*) indicates that the demand function of preceding crop *m*, $d_m^s(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu})$, strictly decreases in $\mu_m$ as long as this demand is strictly positive and that this demand is null for a sufficiently large level of $\mu_m$. The term $\mu_m$ can be seen as the renting price of a unit of land with preceding crop *m*. In the next section it denotes the Lagrange multiplier associated to the crop rotation constraint $d_{m,t-1} = \boldsymbol{\iota}'\mathbf{s}_{(m),t} = \boldsymbol{\iota}'\mathbf{s}_{m,t-1} = a_{m,t-1}$.

## 5. Myopic acreage choices

The considered farmer solves the following problem:

(M) $\max_{\mathbf{s} \geq \mathbf{0}} \{\Pi(\mathbf{s};\boldsymbol{\pi}) \text{ s.t. } \mathbf{Ds} = \mathbf{a}\}$ where $\mathbf{s} \in \mathbb{R}_+^N$, $\boldsymbol{\pi} \in \mathbb{R}^N$ and $\mathbf{a} \in \mathcal{A} \equiv \{\mathbf{a} \in \mathbb{R}_+^K : \boldsymbol{\iota}'\mathbf{a} > 0\}$

if he adopts a myopic strategy. He accounts for the effects of his past choices, **a**, on his current choices, **s**, but he ignores the fact that his current choice will affect his future choice

collecting the separating frontiers of the polyhedral partition $\{\mathcal{P}_j : j \in \mathcal{J}\}$. This last set has null Lebesgue measure (almost everywhere).

[10] Jittorntrum's (1984) results provide an extension to the classical Envelope Theorem and a partial extension to the Implicit Function Theorem (see, *e.g*., Fiacco and Kyparisis, 1985).

opportunities. The investigation of the solutions to this problem is useful for analyzing the mechanisms underlying the effects of the crop rotation constraints $\mathbf{Ds}=\mathbf{a}$ on the acreage choices as well as on the properties of its value functions.

The results characterizing the solution functions to problem (M) are collected in Proposition 2.

**Proposition 2**. *Myopic problem*. Let consider the problem $\max_{\mathbf{s}\geq\mathbf{0}}\{\Pi(\mathbf{s};\boldsymbol{\pi})\text{ s.t. }\mathbf{Ds}=\mathbf{a}\}$ under the assumptions defined in Proposition 1. Let further assume that $\mathbf{a}\in\mathcal{A}\equiv\{\mathbf{a}\in\mathbb{R}_{+}^{K}:\boldsymbol{\iota}'\mathbf{a}>0\}$.

(*i*) The solution in **s** to problem (M) is unique and defines the function $\mathbf{s}^{m}:\mathbb{R}^{N}\times\mathcal{A}\rightarrow\mathbb{R}_{+}^{N}$ by $\mathbf{s}^{m}(\boldsymbol{\pi},\mathbf{a})\equiv\arg\max_{\mathbf{s}\geq\mathbf{0}}\{\Pi(\mathbf{s};\boldsymbol{\pi})\text{ s.t. }\mathbf{Ds}=\mathbf{a}\}$. $\mathbf{s}^{m}$ is continuous in $(\boldsymbol{\pi},\mathbf{a})$ on $\mathbb{R}^{N}\times\mathcal{A}$.

(*ii*) The solution to problem (M) defines the value function $\Pi^{m}:\mathbb{R}^{N}\times\mathcal{A}\rightarrow\mathbb{R}$ by $\Pi^{m}(\boldsymbol{\pi},\mathbf{a})\equiv\max_{\mathbf{s}\geq\mathbf{0}}\{\Pi(\mathbf{s};\boldsymbol{\pi})\text{ s.t. }\mathbf{Ds}=\mathbf{a}\}$. $\Pi^{m}$ is convex in $\boldsymbol{\pi}$ and concave in **a** on $\mathbb{R}^{N}\times\mathcal{A}$.

(*iii*) $\mathbf{s}^{m}$ is piecewise affine and $\Pi^{m}$ is piecewise quadratic in $(\boldsymbol{\pi},\mathbf{a})$ on $\mathbb{R}^{N}\times\mathcal{A}$.

(*iv*) $\Pi^{m}$ is continuously differentiable in $(\boldsymbol{\pi},\mathbf{a})$ on $\mathbb{R}^{N}\times\mathcal{A}$ with $\frac{\partial}{\partial\boldsymbol{\pi}}\Pi^{m}(\boldsymbol{\pi},\mathbf{a})=\mathbf{s}^{m}(\boldsymbol{\pi},\mathbf{a})$ and $\boldsymbol{\mu}^{m}(\boldsymbol{\pi},\mathbf{a})\equiv\frac{\partial}{\partial\mathbf{a}}\Pi^{m}(\boldsymbol{\pi},\mathbf{a})$.

Let define the Lagrangian problem associated to problem (M) as:

(LM) $\min_{\boldsymbol{\mu},\boldsymbol{\lambda}\geq\mathbf{0}}\max_{\mathbf{s}}\{\Pi(\mathbf{s};\boldsymbol{\pi})+\mathbf{s}'\boldsymbol{\lambda}+\boldsymbol{\mu}'(\mathbf{a}-\mathbf{Ds})\}$

and let $\mathcal{M}^{m}(\boldsymbol{\pi},\mathbf{a})$ denote the set of solutions in $\boldsymbol{\mu}$ to problem (LM).

($v$) $\mathbf{s}^s(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})=\mathbf{s}^m(\boldsymbol{\pi},\mathbf{a})$ and $\Pi^s(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})+\boldsymbol{\mu}'\mathbf{a}=\Pi^m(\boldsymbol{\pi},\mathbf{a})$ if and only if $\boldsymbol{\mu}\in\mathcal{M}^m(\boldsymbol{\pi},\mathbf{a})$.

($vi$) $\mathcal{M}^m(\boldsymbol{\pi},\mathbf{a})=\arg\min_{\boldsymbol{\mu}\in\mathbb{R}^K}\{\Pi^s(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})+\boldsymbol{\mu}'\mathbf{a}\}$ and $\mathcal{M}^m(\boldsymbol{\pi},\mathbf{a})=\{\boldsymbol{\mu}\in\mathbb{R}^K:\mathbf{D}\mathbf{s}^s(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})=\mathbf{a}\}$.

($vii$) Let define the sets $\mathcal{K}^+(\mathbf{a})\equiv\{n\in\mathcal{K}:a_n>0\}$ and $\mathcal{K}^0(\mathbf{a})\equiv\{n\in\mathcal{K}:a_n=0\}$.

($a$) If $n\in\mathcal{K}^+(\mathbf{a})$ then the solutions in $\boldsymbol{\lambda}_{(n)}$ and $\mu_n$ to problem (LM) are unique. Let denote these solutions by $\boldsymbol{\lambda}^m_{(n)}(\boldsymbol{\pi},\mathbf{a})$ and $\mu^m_n(\boldsymbol{\pi},\mathbf{a})$.

($b$) If $n\in\mathcal{K}^0(\mathbf{a})$ then there exists $\mu^m_n(\boldsymbol{\pi},\mathbf{a})>-\infty$ such that $\mu^m_n$ is a solution in $\mu_n$ to problem (LM) if and only if $\mu^m_n\in[\mu^m_n(\boldsymbol{\pi},\mathbf{a}),+\infty)$.

($c$) $\frac{\partial}{\partial a_n}\Pi^m(\boldsymbol{\pi},\mathbf{a})=\mu^m_n(\boldsymbol{\pi},\mathbf{a})$ and $\boldsymbol{\mu}^m(\boldsymbol{\pi},\mathbf{a})\equiv(\mu^m_n(\boldsymbol{\pi},\mathbf{a}):n\in\mathcal{K})$ for any $(\boldsymbol{\pi},\mathbf{a})\in\mathbb{R}^N\times\mathcal{A}$.

**Proof**. See Technical Appendix.

Results ($i$)-($iii$) either are well known or are direct applications of the results of Bemporad *et al* (2002). The continuous differentiability of $\Pi^m$ in $(\boldsymbol{\pi},\mathbf{a})$ on $\mathbb{R}^N\times\mathcal{A}$ are less standard results which are obtained by applying results due to Berkelaar *et al* (1997) as well as the results of Jittorntrum (1984) mentioned above. The differentiability of $\Pi^m$ in $\mathbf{a}$ is an essential property of the value function $\Pi^m$. In particular, the first derivative of $\Pi^m(\boldsymbol{\pi},\mathbf{a})$ in **a**, *i.e.*:

$$\frac{\partial}{\partial\mathbf{a}}\Pi^m(\boldsymbol{\pi},\mathbf{a})=\boldsymbol{\mu}^m(\boldsymbol{\pi},\mathbf{a})\text{ for any }(\boldsymbol{\pi},\mathbf{a})\in\mathbb{R}^N\times\mathcal{A},\tag{10}$$

has both a very useful economic interpretation and a central role for characterizing the solutions to the dynamic programming problem (D). The main difficulty in the proof of these properties of

$\mathbf{s}^m$ and $\Pi^m$ lies in the cases where $a_n = 0$ for some $n \in \mathcal{K}$. These cases cannot be excluded as they are likely to frequently occur in applications. They are considered in result (*vii*).

The term $\boldsymbol{\lambda} \in \mathbb{R}_+^N$ denotes the Lagrange multiplier vector associated to the non-negativity constraint $\mathbf{s} \geq \mathbf{0}$ while $\boldsymbol{\mu} \in \mathbb{R}^K$ denotes the Lagrange multiplier vector associated to the crop rotation constraint $\mathbf{Ds} = \mathbf{a}$. The Lagrangian problem (LM) defined in Proposition 2 is equivalent to the following modified[11] Lagrangian problem:

$$\min_{\boldsymbol{\mu}} \max_{\mathbf{s}\geq\mathbf{0}} \{\Pi(\mathbf{s};\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a}\}. \qquad (11)$$

Observing that the elements of $\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}$ are given by $\pi_{nk} - \mu_n$, the acreage choice problem defined by $\max_{\mathbf{s}\geq\mathbf{0}} \Pi(\mathbf{s};\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu})$ has a simple interpretation. This virtual problem is defined as if markets for land with specific preceding crops are actually available. The renting prices of the land unit with specific preceding crops are given in $\boldsymbol{\mu}$. The farmer can rent land with specific preceding crops, at total gross cost equals to $\boldsymbol{\mu}'\mathbf{Ds}$, while having the opportunity to lend his own plots according to their last crop, with a total gross revenue equals to $\boldsymbol{\mu}'\mathbf{a}$.

Besides this useful interpretation of the crop rotation Lagrange multipliers, the problem given in equation (11) also provides the first part of the link between problem (M) and problem (S). According to Proposition 1 the problem given in equation (11) is equivalent to the dual problem:

$$\min_{\boldsymbol{\mu}} \{\Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a}\} \qquad (12)$$

[11] since keeping implicit the non-negativity constraint $\mathbf{s} \geq \mathbf{0}$.

with:

$$\tfrac{\partial}{\partial \boldsymbol{\pi}} \Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) = \mathbf{s}^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) = \arg\max_{\mathbf{s} \geq \mathbf{0}} \Pi(\mathbf{s}; \boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}). \qquad (13)$$

It suffices to observe that $\max_{\mathbf{s} \geq \mathbf{0}} \Pi(\mathbf{s}; \boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) = \Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu})$. Result (*v*) states that $\mathbf{s}^m(\boldsymbol{\pi}, \mathbf{a})$ and $\Pi^m(\boldsymbol{\pi}, \mathbf{a})$ are equal to $\mathbf{s}^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu})$ and $\Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a}$ when $\boldsymbol{\mu}$ is a solution to the dual problem described in equation (12), *i.e.* when $\boldsymbol{\mu} \in \arg\min_{\boldsymbol{\mu}} \{\Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a}\} = \mathcal{M}^m(\boldsymbol{\pi}, \mathbf{a})$.

The second part of the link between problems (M) and (S) is provided by result (*vi*). This result states that $\boldsymbol{\mu} \in \mathcal{M}^m(\boldsymbol{\pi}, \mathbf{a})$ if and only if $\boldsymbol{\mu}$ is a solution to the equation $\mathbf{D}\mathbf{s}^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) = \mathbf{a}$ , *i.e.* if and only if $\boldsymbol{\mu}$ "enforces" the crop rotation constraints $\mathbf{D}\mathbf{s} = \mathbf{a}$.

The elements of the vector $\mathbf{D}\mathbf{s}^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu})$ can be interpreted as the demand functions for land with specific previous crops where the elements of $\boldsymbol{\mu}$ are the renting prices of these differentiated land areas. These demand functions describe derive demands mainly driven by the current profitability of the different crop sequences. *E.g.*, the demand of the land with the best previous crops of crop *k* is high if assuming that the price of crop *k* is high. This intuitive result is a consequence of result (*vi*) of Proposition 1.

The availability of markets for land with specific preceding crops is a potential source of additional profit, at any renting/lending price $\boldsymbol{\mu}$. The standard duality inequality related to the Lagrangian problem (11) yields:

$$\Pi^s(\bar{\boldsymbol{\pi}} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a} \geq \Pi^m(\boldsymbol{\pi}, \mathbf{a}) = \min_{\boldsymbol{\mu}} \{\Pi^s(\boldsymbol{\pi} - \mathbf{D}'\boldsymbol{\mu}) + \boldsymbol{\mu}'\mathbf{a}\}. \qquad (14)$$

The increase in profit due to the availability of "complete" land markets rely on two mechanisms. First, the considered farmer can adjust his total crop area, *i.e.* $\boldsymbol{\iota}'\mathbf{D}\mathbf{s}^{s}(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})\neq\boldsymbol{\iota}'\mathbf{a}$ in general. He can increase his total crop area if $\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu}$ is "high" and he can decrease it if $\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu}$ is "low". Second, the farmer can rent and lend as much land with specific preceding crops as he wishes to, according to the comparison of the elements of $\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu}$. In this case farmers' choices are only constrained by the acreage management costs.

At any $\boldsymbol{\mu}\in\mathcal{M}^{m}(\boldsymbol{\pi},\mathbf{a})$, the farmer rents from himself his past acreage and, consequently, do not lend any piece of land. Any land price vector $\boldsymbol{\mu}$ in $\mathcal{M}^{m}(\boldsymbol{\pi},\mathbf{a})$ basically eliminates the additional benefits from the availability of markets for land with specific preceding crops.

It remains to interpret the marginal effect of $\mathbf{a}$ on $\Pi^{m}(\boldsymbol{\pi},\mathbf{a})$, *i.e.* the term $\boldsymbol{\mu}^{m}(\boldsymbol{\pi},\mathbf{a})$. According to result (*vii*) $\boldsymbol{\mu}^{m}(\boldsymbol{\pi},\mathbf{a})$ is an equilibrium price vector of the land markets for land with specific crop prices, *i.e.* $\boldsymbol{\mu}^{m}(\boldsymbol{\pi},\mathbf{a})$ is a solution to the equation $\mathbf{D}\mathbf{s}^{s}(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})=\mathbf{a}$. However, this equation has infinitely many solutions in $\boldsymbol{\mu}\in\mathbb{R}^{K}$ if $a_{n,t-1}=0$ for some $n\in\mathcal{K}$, *i.e.* if a previous crop is not available to the farmer. As shown by result (*vii*) of Propositions 1 and 2, the constraint $\boldsymbol{\iota}'\mathbf{s}_{(n)}^{s}(\boldsymbol{\pi}-\mathbf{D}'\boldsymbol{\mu})=a_{n}=0$ can be enforced by choosing $\mu_{n}$ sufficiently large.[12] Hence $\boldsymbol{\mu}^{m}(\boldsymbol{\pi},\mathbf{a})$ is just a particular element of $\mathcal{M}^{m}(\boldsymbol{\pi},\mathbf{a})$.

[12] From a technical viewpoint, the multiplicity of the solutions in $(\boldsymbol{\lambda}_{(n)},\mu_{n})$ for $n\in\mathcal{K}$ such that $a_{n}=0$ comes from the fact that the non-negativity constraints $\mathbf{s}_{(n)}\geq\mathbf{0}$ and the crop rotation constraint $\boldsymbol{\iota}'\mathbf{s}_{(n)}\geq a_{n}$ are redundant if $a_{n}=0$, with $\mathbf{s}_{(n)}=\mathbf{0}$.

According to result (*vii*) of Proposition 2 the term $\mu_n^m(\boldsymbol{\pi},\mathbf{a})$ is the lowest equilibrium price of the "farm market" for preceding crop *n*. It is also the price of land with preceding crop *n* which implies that the farmer's demand for such land is exactly equal to 0 if $a_{n,t-1}=0$. Any decrease in $\mu_n$ from $\mu_n^m(\boldsymbol{\pi},\mathbf{a})$ increases the demand for land with preceding crop *n* to strictly positive levels. In fact, the optimal Lagrange multiplier $\mu_n^m(\boldsymbol{\pi},\mathbf{a})$ of the crop rotation constraint *n* can be interpreted as the maximal willingness to pay for renting a unit of land with preceding crop *n* of the farmer, whether such land is available on his farm or not. This is the usual interpretation of the marginal effect of a constraint parameter on the value function of a maximization problem.

## 6. Dynamic acreage choices

The main theoretical tools which are to be used in this section for characterizing the solution to problem (D) have been presented in the previous sections. They are employed in this section for analyzing a specific, albeit fairly general, version of problem (D).

We now assume that the probability distribution of $\tilde{\omega}_t$ conditional on $\omega_1$ is discrete with finite support. Of course, this assumption is restrictive. But the support sets $\mathcal{W}_t$ of the exogenous state variables $\tilde{\omega}_t$ can be defined as large as one wants in our theoretical analysis. Moreover, in the case where $\tilde{\omega}_t$ is assumed to be continuous, any solution approach to problem (D) is likely to rely on some discretization of the continuous support of $\mathcal{W}_t$. In that case the set $\mathcal{W}_T$ can be interpreted as the scenario set of the considered problem. It contains the $W_T \equiv \#\mathcal{W}_T$ possible histories for the exogenous state variables of the problem from date 1 to date *T*. Similarly, $\mathcal{W}_t$ is the set containing the $W_t \equiv \#\mathcal{W}_t$ sub-scenarios from date 1 to date *t*.

Let $p_{\omega_t} \in (0,1)$ denote the probability conditional on $\omega_1$ of the event $\tilde{\omega}_t = \omega_t$. Of course, we have $\sum_{\omega_t \in \mathcal{W}_t} p_{\omega_t} = 1$ and $p_{\omega_1} = 1$. Let also $\mathcal{W}_t(\omega_{t-1})$ denote the support of the probability distribution of $\tilde{\omega}_t$ conditional on $\omega_{t-1}$ and, finally, let $p_{\omega_t|\omega_{t-1}} \in (0,1)$ denote the probability conditional on $\omega_{t-1}$ of the event $\tilde{\omega}_t = \omega_t$. The definition of the $\omega_t$ terms as information sets implies a nested structure of the supports $\mathcal{W}_t$ : $\{\mathcal{W}_t(\omega_{t-1}) : \omega_{t-1} \in \mathcal{W}_{t-1}\}$ defines a mutually exclusive partition of $\mathcal{W}_t$. This implies that $p_{\omega_t|\omega_{t-1}} = p_{\omega_t}(p_{\omega_{t-1}})^{-1}$ for any $\omega_t \in \mathcal{W}_t$. If $\omega_t \in \mathcal{W}_t(\omega_{t-1})$ then $\omega_{t-1}$ is the sub-scenario from date 1 to date $t-1$ of the scenario from date 1 to date *t* described by $\omega_t$.

The probability distributions of the control variable $\tilde{\mathbf{s}}_t$ can be defined based on those of $\tilde{\omega}_t$. The random variable $\tilde{\mathbf{s}}_t$ conditional on $\omega_1$ is defined as the discrete random variable characterized by the support $\{\mathbf{s}_{\omega_t} : \omega_t \in \mathcal{W}_t\}$ and by the probability set of $\tilde{\omega}_t$, *i.e.* by $\{p_{\omega_t} : \omega_t \in \mathcal{W}_t\}$. Of course, to compute the solutions in **s** of problem (D) consists in computing the optimal levels of the support points $\mathbf{s}_{\omega_t}$ for $\omega_t \in \mathcal{W}_t$.

These notations allow rewriting problem (D) as problem (D$^d$):

$$(\mathrm{D}^d) \quad \max_{\mathbf{s}\geq\mathbf{0}} \left\{ \begin{array}{c} \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t \in \mathcal{W}_t} p_{\omega_t} \times \Pi(\mathbf{s}_{\omega_t}; \bar{\boldsymbol{\pi}}_{t|\omega_t}) \\ \text{s.t.} \\ \mathbf{D}\mathbf{s}_{\omega_1} = \mathbf{a}_0 \\ \mathbf{D}\mathbf{s}_{\omega_t} = \mathbf{A}\mathbf{s}_{\omega_{t-1}} \text{ for } \omega_t \in \mathcal{W}_t(\omega_{t-1}),\ \omega_{t-1} \in \mathcal{W}_{t-1} \text{ and } t = 2,...,T \end{array} \right\}$$

where $\mathbf{s} \equiv (\mathbf{s}_{(t)} : t = 1,...,T)$ and $\mathbf{s}_{(t)} \equiv (\mathbf{s}_{\omega(t)} : \omega_t \in \mathcal{W}_t)$ and with the convention $\mathbf{s}_{\tilde{\omega}_1} \equiv \mathbf{s}_{\omega_1}$. The solution in $\mathbf{s}$ to problem (D$^d$) can be defined as functions of $\mathbf{a}_0$ and $\bar{\boldsymbol{\pi}} \equiv (\bar{\boldsymbol{\pi}}_{(t)} : t = 1,...,T)$ where $\bar{\boldsymbol{\pi}}_{(t)} \equiv (\bar{\boldsymbol{\pi}}_{t|\omega_t} : \omega_t \in \mathcal{W}_t)$.

Of course problem (D$^d$) may be seriously affected by the curse of dimensionality. The number of support points to be computed, *i.e.* $W \equiv 1 + \sum_{t=2}^{T} W_t$, may be huge. $W$, the number of scenarios and sub-scenarios to be considered, depends on $T$ and on $W_t$ for $t = 2,...,T$. Each support point of $\tilde{\mathbf{s}}_t$ contains $N = K^2$ elements and that $W_t$, the number of support points of $\tilde{\mathbf{s}}_t$, grows exponentially in $t$.

The support of the exogenous state variable being discrete and finite, two approaches are available for solving problem (D$^d$). The stochastic programming approach consists in directly solving problem (D$^d$) in $\mathbf{s}$ while taking some advantage of its structure.[13] In fact problem (D$^d$) can be interpreted as a (very) large "static" quadratic programming problem with a specific multistage structure. The dynamic programming approach relies on Bellman's dynamic principle. It involves recursively defined value (or recourse) functions. In the dynamic programming approach, the optimal values of the $\mathbf{s}_{(t)}$ terms are characterized as functions of the state variables based on a backward recursion while they are characterized simultaneously for $t = 1,...,T$ in the

[13] One such approach consists in optimally computing $W_T$ sequences of acreage choices, one for each scenario of $\mathcal{W}_T$. Of course, the optimal sequences cannot be independently computed. The acreage choice sequence up to date $t$ of two scenarios which are identical up to date $t$ must be equal, *i.e.* the so-called non-anticipatory constraints need to be enforced.

stochastic programming approach. In what follows we consider both approaches as they emphasize different features of the considered dynamic stochastic optimization problem.

The stochastic programming approach consists in considering problem ($\mathrm{D}^d$) as a (very) large quadratic programming problem involving the objective function:

$$\sum_{t=1}^{T}\beta^{t-1}\sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t}\Pi(\mathbf{s}_{\omega_t};\bar{\boldsymbol{\pi}}_{t|\omega_t}) \tag{15}$$

and the non-empty feasible set:

$$\mathcal{F}(\mathbf{a}_0)\equiv\{\mathbf{s}\in\mathbb{R}_+^{NW}:\mathbf{D}\mathbf{s}_{\omega_1}=\mathbf{a}_0 \text{ and } \mathbf{D}\mathbf{s}_{\omega_t}=\mathbf{A}\mathbf{s}_{\omega_{t-1}} \text{ for } \omega_t\in\mathcal{W}_t(\omega_{t-1}),\ \omega_{t-1}\in\mathcal{W}_{t-1},\ t=2,...,T\}. \tag{16}$$

The objective function of problem ($\mathrm{D}^d$) is strictly concave in the control variable **s** and the feasible set $\mathcal{F}(\mathbf{a}_0)$ has a non-empty interior, implying that problem ($\mathrm{D}^d$) is strongly dual. Proposition 3 provides results for characterizing the solutions to problem ($\mathrm{D}^d$) using a stochastic programming approach.

**Proposition 3**. *Dynamic problem*. Let consider the stochastic dynamic optimization problem ($\mathrm{D}^d$): $\max_{\mathbf{s}\in\mathcal{F}(\mathbf{a}_0)}\sum_{t=1}^{T}\beta^{t-1}\sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t}\Pi(\mathbf{s}_{\omega_t};\bar{\boldsymbol{\pi}}_{t|\omega_t})$ where the feasible set $\mathcal{F}(\mathbf{a}_0)$ is defined in equation (16) and under the assumptions stated in Propositions 1 and 2.

(*i*) The solution in **s** to problem ($\mathrm{D}^d$) is unique. Let $\mathbf{s}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ denote this solution with

$\mathbf{s}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0)\equiv(\mathbf{s}^o_{(t)}(\bar{\boldsymbol{\pi}},\mathbf{a}_0):t=1,...,T)$ and $\mathbf{s}^o_{(t)}(\bar{\boldsymbol{\pi}},\mathbf{a}_0)\equiv(\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}},\mathbf{a}_0):\omega_t\in\mathcal{W}_t)$.

(*ii*) The function $\mathbf{s}^o:\mathbb{R}^{NW}\times\mathcal{A}\to\mathbb{R}_+^{NW}$ defined by the solution in **s** to problem ($\mathrm{D}^d$):

$$\mathbf{s}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0) \equiv \arg\max_{\mathbf{s}\in\mathcal{F}(\mathbf{a}_0)} \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t} \Pi(\mathbf{s}_{\omega_t};\bar{\boldsymbol{\pi}}_{t|\omega_t}) .$$

is piecewise affine and continuous in $(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ on $\mathbb{R}^{NW}\times\mathcal{A}$.

(*iii*) The value function $V_{\omega_1}^o:\mathbb{R}^{NW}\times\mathcal{A}\to\mathbb{R}^{NW}$ of problem (D$^d$) defined by:

$$V_{\omega_1}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0) \equiv \max_{\mathbf{s}\in\mathcal{F}(\mathbf{a}_0)} \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t} \Pi(\mathbf{s}_{\omega_t};\bar{\boldsymbol{\pi}}_{t|\omega_t})$$

is piecewise quadratic and continuous in $(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ on $\mathbb{R}^{NW}\times\mathcal{A}$. $V_{\omega_1}^o$ is convex in $\bar{\boldsymbol{\pi}}$ and concave in $\mathbf{a}_0$ on $\mathbb{R}^{NW}\times\mathcal{A}$.

Let $\boldsymbol{\mu}\in\mathbb{R}^{KW}$ denote a vector with the same structure as **s**, *i.e.* $\boldsymbol{\mu}\equiv(\boldsymbol{\mu}_{(t)}:t=1,...,T)$ with $\boldsymbol{\mu}_{(t)}\equiv(\boldsymbol{\mu}_{\omega_t}:\omega_t\in\mathcal{W}_t)$. Let define the term $\bar{\boldsymbol{\mu}}_{t+1|\omega_t}$ by $\bar{\boldsymbol{\mu}}_{t+1|\omega_t}\equiv\sum_{\omega_{t+1}\in\mathcal{W}_{t+1}(\omega_t)} p_{\omega_{t+1}|\omega_t}\boldsymbol{\mu}_{\omega_{t+1}}$ with the convention $\boldsymbol{\mu}_{\omega_{T+1}}\equiv\mathbf{0}$.

(*iv*) To solve problem (D$^d$) is equivalent to solve the following dual problem:

(DD$^d$) $\quad \min_{\boldsymbol{\mu}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t} \Pi^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) + \boldsymbol{\mu}'_{\omega_1}\mathbf{a}_0 \right\}$

with:

$\mathbf{s}_{\omega_t}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0) = \mathbf{s}^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t})$ for $\omega_t\in\mathcal{W}_t$ and $t=1,...,T$

for any $\boldsymbol{\mu}\in\mathcal{M}_{\omega_1}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ where $\mathcal{M}_{\omega_1}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ is the solution set in to problem (DD$^d$), *i.e.*:

$$\mathcal{M}_{\omega_1}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0) \equiv \arg\min_{\boldsymbol{\mu}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t\in\mathcal{W}_t} p_{\omega_t} \Pi^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) + \boldsymbol{\mu}'_{\omega_1}\mathbf{a}_0 \right\} .$$

(*v*) $\boldsymbol{\mu}\in\mathcal{M}_{\omega_1}^o(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$ if and only if $\boldsymbol{\mu}$ is a solution to equation system:

$$\begin{cases} \mathbf{Ds}^s(\bar{\boldsymbol{\pi}}_{1|\omega_1} - \mathbf{D}'\boldsymbol{\mu}_{\omega_1} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{2|\omega_1}) = \mathbf{a}_0 \\ \mathbf{Ds}^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) = \mathbf{As}^s(\bar{\boldsymbol{\pi}}_{t-1|\omega_{t-1}} - \mathbf{D}'\boldsymbol{\mu}_{\omega_{t-1}} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t|\omega_{t-1}}) \\ \text{for } \omega_{t-1} \in \mathcal{W}_{t-1},\ \omega_t \in \mathcal{W}_t(\omega_{t-1}) \text{ and } t = 2,\ldots,T. \end{cases}$$

(*vi*) $V^o_{\omega_1}$ is continuously differentiable in $\boldsymbol{\pi}$ on $\mathbb{R}^{NW} \times \mathcal{A}$ with $\frac{\partial}{\partial\boldsymbol{\pi}} V^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0) = \mathbf{s}^o(\bar{\boldsymbol{\pi}}, \mathbf{a}_0)$.

(*vii*) $V^o_{\omega_1}$ is continuously differentiable in $\mathbf{a}_0$ on $\mathbb{R}^{NW} \times \mathcal{A}$, *i.e*. there exists a function

$\boldsymbol{\mu}^o_{\omega_1} : \mathbb{R}^{NW} \times \mathcal{A} \to \mathcal{M}^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0)$, continuous in $\mathbf{a}_0$ on $\mathbb{R}^{NW} \times \mathcal{A}$, such that $\frac{\partial}{\partial\mathbf{a}_0} V^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0) = \boldsymbol{\mu}^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0)$.

**Proof**. See Technical Appendix.

Once again results (*i*)-(*iii*) obtained are by applying the results obtained by Bemporad *et al* (2002). It suffices to observe that problem ($D^d$) is a quadratic programming problem. Result (*iv*) is demonstrated by considering a specific version of the Lagrangian problem associated to problem ($D^d$) where $\boldsymbol{\mu} \in \mathbb{R}^{NW}$ is the vector collecting the Lagrange multiplier vectors associated to the crop rotation constraints arising in each possible sub-scenario of the exogenous state variable $\tilde{\omega}_t$ for $t = 1,\ldots,T$ (see the Technical Appendix).[14] This Lagrangian problem is equivalent to the following modified Lagrangian problem:

$$\min_{\boldsymbol{\mu}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t \in \mathcal{W}_t} p_{\omega_t} \max_{\mathbf{s}_{\omega_t} \geq \mathbf{0}} \Pi(\mathbf{s}_{\omega_t}; \bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) + \boldsymbol{\mu}'_{\omega_1}\mathbf{a}_0 \right\}. \quad (17)$$

The objective function of this problem is a discounted sum of expected static maximization problems interrelated *via* the crop rotation constraints Lagrange multiplier vectors collected in $\boldsymbol{\mu}$.

[14] Indeed, this formulation of the Lagrangian problem associated to problem ($D^d$) simply takes advantage of the multistage structure of this problem, as in the dynamic programming approach.

In this objective function, the considered farmer solves a static acreage choice problem for any possible sub-scenario of the exogenous state variable $\tilde{\omega}_t$ for $t=1,...,T$. In each of this sub-scenario, he solves the profit maximization problem:

$$\max_{\mathbf{s}_{\omega_t}\geq \mathbf{0}} \Pi(\mathbf{s}_{\omega_t};\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}), \tag{18}$$

*i.e.* the farmer has the opportunity to rent and lend land on virtual (contingent) markets at prices defined by $\boldsymbol{\mu}$. The elements of the vector $\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}$ can be interpreted as "crop sequence dynamic gross margins". They are defined by:

$$\bar{\pi}_{mk,t|\omega_t} - \mu_{m,\omega_t} + \beta\bar{\mu}_{k,t+1|\omega_t} \text{ where } \bar{\mu}_{k,t+1|\omega_t} \equiv \sum_{\omega_{t+1}\in\mathcal{W}_{t+1|\omega_t}} p_{\omega_{t+1}|\omega_t}\mu_{k,\omega_{t+1}}. \tag{19}$$

According to problem (.) and to the "dynamic gross margins" defined in the previous equation, farmers choose their acreage in the sub-scenario $\omega_t$ by maximizing their current farm profit with the opportunity to rent land with specific preceding crops at price $\boldsymbol{\mu}_{\omega_t}$ and knowing that they will have the opportunity to lend their current acreage as land with specific preceding crops in the next period. They anticipate to lend their acreage $\mathbf{As}_t$ at expected price $\bar{\boldsymbol{\mu}}_{t+1|\omega_t}$ and the resulting discounted expected gross revenue is given by $\beta\bar{\boldsymbol{\mu}}'_{t+1|\omega_t}\mathbf{As}_t$. Basically, the terms $\boldsymbol{\mu}_{\omega_t}$ and $\beta\bar{\boldsymbol{\mu}}_{t+1|\omega_t}$ sum up the dynamic features of the dynamic acreage choice process.

The feasible set $\mathcal{F}(\mathbf{a}_0)$ having an non-empty interior the modified Lagrangian problem given in equation (17) has solutions in $\boldsymbol{\mu}$ and these solutions, which define the set $\mathcal{M}^o_{\omega_1}(\bar{\boldsymbol{\pi}},\mathbf{a}_0)$, allows characterizing the solutions to problem ($\text{D}^d$). By Proposition 1, the problem given in equation (17) is equivalent to the problem:

$$\min_{\boldsymbol{\mu}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t \in \mathcal{W}_t} p_{\omega_t} \Pi^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta \mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) + \boldsymbol{\mu}'_{\omega_1}\mathbf{a}_0 \right\} \quad (20)$$

and this yields:

$$V^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0) = \min_{\boldsymbol{\mu}} \left\{ \sum_{t=1}^{T} \beta^{t-1} \sum_{\omega_t \in \mathcal{W}_t} p_{\omega_t} \Pi^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta \mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) + \boldsymbol{\mu}'_{\omega_1}\mathbf{a}_0 \right\} \quad (21)$$

and:

$$\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0) = \mathbf{s}^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta \mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t}) \text{ for } \omega_t \in \mathcal{W}_t \text{ and } t = 1,...,T \text{ if } \boldsymbol{\mu} \in \mathcal{M}^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0). \quad (22)$$

Note that the minimization problem stated in equation (21) can be interpreted as a (modified) dual problem associated to the (primal) problem ($D^d$).

Result (*iv*) states that the solution set in $\boldsymbol{\mu}$ to the problem given in (20), $\mathcal{M}^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0)$, is characterized as the subset of $\mathbb{R}^{NK}$ containing the vectors $\boldsymbol{\mu}$ which enforce the crop rotation constraint in each possible sub-scenario described by $\omega_t$ for $\omega_t \in \mathcal{W}_t$ and $t = 1,...,T$. The demand of land with specific preceding crops is given by $\mathbf{Ds}^s(\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}_{\omega_t} + \beta \mathbf{A}'\bar{\boldsymbol{\mu}}_{t+1|\omega_t})$ while the term $\mathbf{As}^s(\bar{\boldsymbol{\pi}}_{t-1|\omega_{t-1}} - \mathbf{D}'\boldsymbol{\mu}_{\omega_{t-1}} + \beta \mathbf{A}'\bar{\boldsymbol{\mu}}_{t|\omega_{t-1}})$ (or $\mathbf{a}_0$ if $t = 1$) defines the corresponding land supply, in sub-scenario $\omega_t$. According to our interpretation of $\boldsymbol{\mu}$ as price vectors of market of land with specific preceding crops, any element of $\mathcal{M}^o_{\omega_1}(\bar{\boldsymbol{\pi}}, \mathbf{a}_0)$ is a vector of contingent price vectors which ensures the equilibrium of the set of contingent markets of land with specific preceding crops defined by each sub-scenario $\omega_t$, for $\omega_t \in \mathcal{W}_t$ and $t = 1,...,T$.

Let now consider problem ($D^d$) following to a dynamic programming perspective, *i.e*. by relying on Bellman's dynamic programming principle. Let define the vector $\bar{\boldsymbol{\pi}}^+_{t|\omega_t}$ as $\bar{\boldsymbol{\pi}}^+_{t|\omega_t} \equiv (\bar{\boldsymbol{\pi}}_{t|\omega_t}, \bar{\boldsymbol{\pi}}^+_{(\omega_t)})$

where $\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)} \equiv (\bar{\boldsymbol{\pi}}_{\tau|\omega_\tau} : \omega_\tau \in \mathcal{W}_\tau(\omega_{\tau-1}),\ \tau = t+1,...,T)$ and let define $W_t^+$ as $W_t^+ \equiv \dim \bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}$. Provided that $\mathbf{a}_{t-1} \in \mathcal{A}$ denote the preceding crop acreage at date *t*, the value functions associated to problem (D$^d$) are recursively defined as:

$$V^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, \mathbf{a}_{t-1}) \equiv \max_{\mathbf{s}_{\omega_t} \geq \mathbf{0}} \{\Pi(\mathbf{s}_{\omega_t}; \bar{\boldsymbol{\pi}}_{t|\omega_t}) + \beta \bar{V}^o_{t+1|\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)}, \mathbf{A}\mathbf{s}_{\omega_t}) \text{ s.t. } \mathbf{D}\mathbf{s}_{\omega_t} = \mathbf{a}_{t-1}\} \tag{23}$$

for $\omega_t \in \mathcal{W}_t$ and $t = 1,...,T$ where $\bar{V}^o_{t+1|\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)}, \mathbf{a}_t) \equiv \sum_{\omega_{t+1} \in \mathcal{W}_{t+1}(\omega_t)} p_{\omega_{t+1}|\omega_t} V^o_{\omega_{t+1}}(\bar{\boldsymbol{\pi}}^{+}_{t+1|\omega_{t+1}}, \mathbf{a}_t)$ and with the convention $V^o_{\omega_{T+1}}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_{T+1})}, \mathbf{a}_T) \equiv 0$. Equation (23) is a direct application of Bellman's principle.

Provided that the problems described this equation have the structure of problem (D$^d$), the results collected in Proposition 3 are applicable. In particular, results (*v*) and (*vi*) provide the differentiability properties of the value functions $V^o_{\omega_t}$. These properties mirror those of $\Pi^m$, the value function of the myopic problem. They are obtained by applying the results of Berkelaar *et al* (1997). The marginal effect of $\bar{\boldsymbol{\pi}}_{t|\omega_t}$ on $V^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, \mathbf{a}_{t-1})$ is summarized by the optimal acreage choice vector $\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, \mathbf{a}_{t-1})$. The value function $V^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, \mathbf{a}_{t-1})$ provides, by definition, the value of crop acreage $\mathbf{a}_{t-1}$ in sub-scenario $\omega_t$, *i.e.* $V^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, \mathbf{a}_{t-1})$ is the discounted sum of the expected optimal profits from date *t* up to date *T* as perceived in sub-scenario $\omega_t$. Result (*vi*) states that the marginal values of the elements of $\mathbf{a}_t = \mathbf{A}\mathbf{s}_{\omega_t}$ in sub-scenario $\omega_t$ are provided by

$$\tfrac{\partial}{\partial \mathbf{a}_t} \bar{V}^o_{t+1|\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)}, \mathbf{A}\mathbf{s}_{\omega_t}) = \bar{\boldsymbol{\mu}}^o_{t+1|\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)}, \mathbf{A}\mathbf{s}_{\omega_t}), \tag{24}$$

*i.e.* a farmer increasing his acreage of crop *k* in the sub-scenario $\omega_t$ by one unit anticipates to "lend to himself" this unit of land at expected price $\bar{\mu}^o_{k,t+1|\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(\omega_t)}, \mathbf{A}\mathbf{s}_{\omega_t})$ in period $t+1$.

The first order conditions in $\mathbf{s}_{\omega_t}$ of the problem defining $V^o_{\omega_t}(\bar{\boldsymbol{\pi}}^{+}_{(t)}, \mathbf{a}_{t-1})$ is given by

$$\begin{aligned}\bar{\boldsymbol{\pi}}_{t|\omega_t} - \mathbf{D}'\boldsymbol{\mu}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{t|\omega_t}, \mathbf{a}_{t-1}) + \beta\mathbf{A}'\bar{\boldsymbol{\mu}}^o_{t+1|\omega_t}\left(\bar{\boldsymbol{\pi}}^+_{(\omega_t)}, \mathbf{A}\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{t|\omega_t}, \mathbf{a}_{t-1})\right) \\ + \boldsymbol{\lambda}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{t|\omega_t}, \mathbf{a}_{t-1}) - \tfrac{\partial}{\partial \mathbf{s}_{\omega_t}} C\left(\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{t|\omega_t}, \mathbf{a}_{t-1})\right) = \mathbf{0}\end{aligned} \tag{25}$$

where $\boldsymbol{\lambda}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{(\omega_t)}, \mathbf{a}_{t-1})$ is the optimal Lagrange multiplier vector associated to the non-negativity constraint $\mathbf{s}_{\omega_t} \geq \mathbf{0}$. These first order conditions provide the formal links between the stochastic programming and dynamic programming approaches. They show that problem ($\mathrm{D}^d$) is of the Euler type in $(\boldsymbol{\mu}, \mathbf{s})$ according to the terminology of Rust (1996). But they also show that the dynamic programming approach doesn't offer much simplification over the stochastic programming approach for solving the finite horizon problem ($\mathrm{D}^d$).[15] Finally, these first order conditions could be used for defining estimating equations in a micro-econometric framework.

## 7. Extensions and implications

The framework presented in this article can be extended for cases where the parameters $(\mathbf{h}, \mathbf{H})$ characterizing the form of the cost function $C$ depend on $\omega_t$, as long as $C$ is a function of $\mathbf{s}_t$ only, as well as to cases where $\mathbf{H}$ is only positive semidefinite. Building on the work of Rockafellar (1987) and of Rockafellar and Wets (1990) on the so-called "extended linear-quadratic framework", Rockafellar (1999) obtained general versions of the main results of Proposition 3. The results presented in Proposition 3, especially those related to the uniqueness of the solution in $\mathbf{s}$ and to the differentiability of the value function, take advantage of the strict convexity of $C$ and of the specificity of the crop rotation constraints.

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[15] Because the functions $\bar{\boldsymbol{\mu}}^o_{t+1|\omega_t}(.)$ or $\bar{V}^o_{t+1|\omega_t}(.)$ are needed for determining the terms $\boldsymbol{\mu}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{(t)}, \mathbf{a}_{t-1})$, $\mathbf{s}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{(t)}, \mathbf{a}_{t-1})$ and $\boldsymbol{\lambda}^o_{\omega_t}(\bar{\boldsymbol{\pi}}^+_{(t)}, \mathbf{a}_{t-1})$.

The extension to cases where $C$ depends on $\omega_t$ is useful for, *e.g.*, introducing risk spreading motives in the analysis. Let assume that the considered farmer is risk averse and, for the ease of exposition, that he is not constrained in his acreage choices by quasi-fixed factor limiting quantities. In a mean-variance approach (see, *e.g.*, Chavas and Holt, 1990 and1996 ; Holt, 1999) the term $C_{\omega_t}(\mathbf{s}_{\omega_t};\theta) \equiv 1/2 \times \theta \times \mathbf{s}_t'(V_{\omega_t}\tilde{\boldsymbol{\pi}}_t)\mathbf{s}_{\omega_t}$ defines the risk premium associated to the random profit $\mathbf{s}_{\omega_t}'\tilde{\boldsymbol{\pi}}_t$. The parameter $\theta$ denotes the absolute risk aversion index summarizing the farmer's attitude toward income risk and $V_{\omega_t}$ denotes the variance operator conditional on $\omega_t$. The per period objective function $E_{\omega_t}\tilde{\boldsymbol{\pi}}_t'\mathbf{s}_{\omega_t} - C_{\omega_t}(\mathbf{s}_{\omega_t};\theta) = E_{\omega_t}\tilde{\boldsymbol{\pi}}_t'\mathbf{s}_{\omega_t} - 1/2 \times \theta \times \mathbf{s}_{\omega_t}'(V_{\omega_t}\tilde{\boldsymbol{\pi}}_t)\mathbf{s}_{\omega_t}$ is the certainty equivalent of $\mathbf{s}_{\omega_t}'\tilde{\boldsymbol{\pi}}_t$ of the considered farmer.

While our framework only considers the effects of the previous crop on the current crop, *i.e.* assumes that the agricultural dynamics is of order 1, agricultural science has shown that crop rotation effects can last for several years. *E.g.* weed seeds, bug eggs or fungus spora may remain viable for several years. The considered framework easily accommodates such increase in the dynamics order, at least in theory. While production dynamics of order 1 requires considering crop sequences of length 2, production dynamics of order 2 requires considering crop sequences of length 3, *etc*. Moreover, there is no need to assume that the same memory length matters for each crop.

However, to account for crop production dynamics of high orders raises practical difficulties of two kinds. High order dynamics can lead to dynamic programming problem with considerable dimension. Dynamic programming problem with uncertainty are already subject to the curse of dimensionality even with order 1 dynamics and short time horizon. Furthermore, the crop rotation effects of long crop sequences are rather poorly documented in the agricultural science literature. In fact our modeling framework is well suited for investigating the effects of crop rotations with relatively short memory.[16]

Of course the presented framework can also be extended to cases where additional constraints, *e.g*. due to limiting quantities of quasi-fixed factors other than land, affect acreage choices. Such an extension would not change the main results presented above as long as the additional constraints are linear in **s**.

LP or PMP models often consider explicit constraints on the activity choices. Some of them impose upper bounds on acreage choices which are justified by crop rotation constraints. Such constraints are often debatable because they rely on arbitrary bounds. In fact they are useless in the present framework. Some crop sequences are impossible due to inconsistencies in biological cycles. These crop sequences can simply be removed from the choice set of problem (D). Crop sequences which are "biologically" possible should be included in the choice set. Their acreage being null or not should result from economic trade-offs. Of course, for some crop sequences the crop rotation effects are so detrimental that they would be profitable only in extreme price

[16] Long memory crop rotation effects usually depend on stock management issues and could be analyzed explicitly as such. It is well known that some nutrients are used in a medium to long run perspective, following an investment strategy aimed at managing soil quality levels. According to the agricultural scientists and extension agents we have consulted, herbicides are often used according to a similar logic. Because some weed species are perennial and the seeds of many weeds are able to survive for many years, the management of weed populations and of weed seed stocks relies on long run strategies.

contexts. *E.g*. the yield level of rapeseed grown after rapeseed is likely to be dramatically low due to pest infestations. Rapeseed after rapeseed acreages can only be profitable in very unlikely situations. The exclusion of this crop sequence from the choice set can be justified according to these economic arguments.

The results of Proposition 3 can also be used for investigating the empirical modeling of farmers' acreage choices. As shown by the econometric analyses cited in the introduction, a dynamic programming approach is more relevant for addressing this issue than the stochastic programming approach adopted for deriving the results of Proposition 3. In fact, according to us the main conclusions to be drawn from the dynamic programming approach deals with the empirical modeling of farmers' acreage choice.

Farmers decide their acreage year after year in an ever evolving context. The observed choice of a farmer with the information set $\omega_t$ and the previous acreage $\mathbf{a}_{t-1}$ can be modeled as the solution in $\mathbf{s}_{\omega_t}$ to the following maximization given in equation (23). This problem accounts for (*i*) the characteristics of the current scenario, *i.e.* $\omega_t$, (*ii*) the features of forward looking behavior, *i.e.* $\beta\bar{V}^{o}_{t+1|\omega_t}$ and $(\bar{\boldsymbol{\pi}}^{+}_{t|\omega_t}, p_{\omega_\tau|\omega_t})$ for $\omega_\tau \in \mathcal{W}_\tau(\omega_{\tau-1})$ and $\tau = t+1,...,T$, and (*iii*) the constraints imposed by past choices, $\mathbf{D}\mathbf{s}_{\omega_t} = \mathbf{a}_{t-1}$. The properties of the expected value function $\bar{V}^{o}_{t+1|\omega_t}$ heavily depend on the probability terms $p_{\omega_\tau|\omega_t}$ for $\omega_\tau \in \mathcal{W}_\tau(\omega_{\tau-1})$ and $\tau = t+1,...,T$, *i.e*. on the anticipations of the considered farmers according to the foregoing economic and bio-physical contexts. This remark is to be related to that of Manski (2004) who basically states that our knowledge on the perceptions of the future by the economic agents usually is very limited despite the crucial role of these perceptions on the current choices of these agents. In particular, Manski (2004) raises an important identification issue: it is difficult to disentangle perceptions of the future and attitudes toward uncertainty by only observing risky choices.

Just and Pope (2003) and Lence (2009) raise other identification problems in agricultural production economics: it is difficult to disentangle attitudes toward uncertainty and technology properties when observing production choices only. Just and Just (2011) supplement the observation of Just and Pope (2003) and Lence (2009) by that of Manski (2004). They consider uncertainty perceptions in addition to attitudes toward uncertainty and technology properties.

By adopting a dynamic perspective, the present study adds a further identification problem in acreage choice models. The value function $\bar{V}^{o}_{t+1|\omega_t}$ is shown to be concave in the current acreage choice $\mathbf{As}_{\omega_t}$. This property of $\bar{V}^{o}_{t+1|\omega_t}$ doesn't rely on the concavity of the profit function $\Pi$ (or, equivalently, on the convexity of the function $C$).[17] It is a direct consequence of dynamic features of the acreage choices considered in problem (D$^d$). This has important implications on the interpretation of standard modeling practices of farmers' acreages choices. A standard acreage choice model adapted from the present framework is usually derived from a static profit maximization problem of the form $\max_{\mathbf{a}_{\omega_t}\geq\mathbf{0}}\{\mathbf{a}'_{\omega_t}\bar{\bar{\boldsymbol{\pi}}}_{t|\omega_t}-C^{a}_{\omega_t}(\mathbf{a}_{\omega_t})\text{ s.t. }\boldsymbol{\iota}'\mathbf{a}_{\omega_t}=\boldsymbol{\iota}'\mathbf{a}_{t-1}\}$ where $\bar{\bar{\boldsymbol{\pi}}}_{t|\omega_t}\equiv(\bar{\bar{\pi}}_{k,t|\omega_t}:k\in\mathcal{K})$ denotes a measure of expected gross margin vector of the considered crop bundle. Notwithstanding the issue related to the aggregation of $\bar{\boldsymbol{\pi}}_{t|\omega_t}$ into $\bar{\bar{\boldsymbol{\pi}}}_{t|\omega_t}$, any statistical estimate or calibration of a term $C^{a}_{\omega_t}(\mathbf{a}_{\omega_t})$ exhibiting concavity in $\mathbf{a}_{\omega_t}$ may capture the effects of a risk premium, of shadow costs of quasi-fixed factor constraints, of a value function accounting

[17] Standard linear programming arguments show that the value function $\bar{V}^{o}_{t+1|\omega_t}$ is concave in $\mathbf{a}_{\omega_t}\equiv\mathbf{As}_{\omega_t}$ if $\mathbf{H}=\mathbf{0}$.

for crop rotation effects and constraints, of partial adjustment costs, or any combination of these crop diversification motives.

## 8. Concluding remarks

Crop rotations are known to have two main kinds of economic effects: direct effects on potential yields and on the productivity of different inputs, and indirect effects on economically optimal input levels, especially pesticides and fertilizers. These effects are summarized in the gross margins of the crop sequences. The main aim of this article is to analyze how these effects affect the acreage choices of forward-looking farmers adopting a farm level strategy.

To consider a farm level strategy instead of a plot level one requires a modeling framework which differs from the often considered dynamic discrete choice model. The main advantage of our approach is that it is more closely related to the models commonly used for empirically investigating farmers' acreage choices, either in the multicrop econometric literature or in the mathematical programming literature. We consider a dynamic programming problem involved per period farm profit with crop rotation constraints. In this modeling framework the crop rotation option values uncovered by the plot level analysis are embedded in the crop rotation constraint Lagrange multipliers.

A main result of this article is that optimal dynamic acreage choices can be formally characterized as static acreage choices with contingent renting/lending markets for acreages with specific preceding crops. The crop rotation constraint Lagrange multipliers provide the renting/lending prices of acreages with specific crop histories.

This market-based interpretation of the crop rotation Lagrange multipliers is not purely theoretical. *E.g.*, in southwestern France corn producers and carrot producers exchange plots for the beneficial effects of alternating crops. The characterization of the dynamic acreage choices provided in Proposition 3 may also be useful for the practical computation of optimal dynamic acreage choices. As suggested by Rockafellar (1999) in a more general framework, this characterization can provide a basis for designing specifically designed algorithms with static acreage choice problems as inner loops and the search for the optimal levels of the crop rotation Lagrange multipliers as outer loop. Such algorithms might be useful for investigating high dimensional optimal dynamic acreage choice through simulation exercises purposes.

The article focuses on the "quadratic case" with finite horizon. But it is worthwhile to note that the theoretical analysis of the solution functions to problems (D) and ($D^d$) rely on general arguments which apply for infinite horizon problems or problem with alternative functional forms for the profit functions and general (linear) constraints on acreage choices.

Quadratic periodic objective functions are common for analyzing dynamic optimization problem in the economic literature. Provided that the considered dynamic optimization problem is a quadratic programming problem, it can be implemented with efficient algorithms coded in any standard optimization software, at least for moderate size problems. The infinite horizon case (with relevant stationary assumptions) is an interesting topic for further research as it can allow characterizing crop rotation schemes aimed at guiding acreage choices.

Although our analysis focuses on the order 1 dynamics case, it provides results which accommodate higher order dynamics, at least in theory. Once again the curse of dimensionality raises practical problems. But, more fundamentally, if our modeling framework is well suited for accounting for short memory crop rotation effects long memory crop rotations effects should probably considered following different approaches.

Besides its purely theoretical aspects, the analysis presented in this article also has implications for applied work on acreage choice modeling, either in the multicrop econometric framework or in the (positive) mathematical programming framework.

The results presented might be used for empirical investigations of farmer's acreage choices. Of course, the empirical investigation of dynamic acreage choices remains a challenge due to data, theoretical and computational issues.

Our results also show that revealed concave effects of acreage in the farmers' objective function might capture the effects of different diversification motives, *i.e*. risk spreading considerations, implicit acreage management costs, partial adjustment costs or the shadow values of crop rotation effects and constraints. These last effects add a further identification issue to those already raised by Just and Pope (2003), Manski (2004), Lence (2009) and Just and Just (2011).

While agricultural economists have mainly focused their efforts on modeling risk spreading and implicit acreage management costs, the increasing concerns related to the impacts of agricultural production on the environment or on public health suggest that more efforts should be put on crop rotation effects and constraints. Crop rotation effects are key elements of most cropping systems aimed at reducing the use of chemical inputs.

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