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Modeling acreage decisions within the multinomial Logit framework

Abstract

The Multinomial Logit (MNL) framework has been used in the agricultural production economics literature to model acreage share choices, crop decisions or land use decisions. This article extends the pioneering works of Caswell and Zilberman (1985) and of Wu and Segerson (1995) by developing further the theoretical background of the MNL acreage share models. Two approaches are considered: the "cost function approach" and the "discrete choice approach". It is then shown that MNL acreage share models can be used to define simple multi-crop econometric models with land as an allocatable fixed input. Finally several generalizations of the standard MNL acreage share model are proposed.

Keywords: acreage share, discrete choice, multicrop econometric model, multinomial Logit

JEL classifications: D21, Q15, C51

Modélisation des choix d'assolements à partir du cadre Logit multinomial

Résumé

La forme Logit multinomial a déjà été utilisée en économie de la production agricole pour spécifier des équations de parts de surface, mais il n'a jamais été démontré que cette forme particulière pouvait être dérivée d'un programme de maximisation du profit. Cet article étend les travaux de Caswell et Zilberman (1985) et de Wu et Segerson (1995), en développant le cadre théorique qui permet de dériver des équations de parts de surface de forme Logit multinomial. Deux approches sont considérées : « l'approche par la fonction de coût » et « l'approche par les choix discrets ». Il est ensuite montré que les parts de surfaces de forme Logit multinomial peuvent être utilisées pour définir un modèle économétrique multi-produit qui considère la terre comme un input fixe allouable. Enfin, plusieurs généralisations de ce modèle sont proposées. En particulier ce type de modèle peut facilement être étendu pour tenir compte des effets des rotations culturales dans les choix d'assolements. Une application empirique est réalisée à partir de données françaises sur des exploitants spécialisés en céréales de 1989 à 2006.

Mots-clefs : choix d'assolement, choix discrets, modèle économétrique multi-produits, Logit multinomial

Classifications JEL : D21, Q15, C51

Modeling acreage decisions within the multinomial Logit framework

1. Introduction

The Multinomial Logit (MNL) framework is mainly used for modeling discrete choices or market shares (McFadden 1974), but its application has also been considered for consumer budget shares (Theil 1969) or cost shares (Considine and Mount 1984). Since the publication of McFadden's (1974) seminal article numerous generalizations of the standard multinomial Logit model have then been introduced in the literature (Train 2003; Ackerberg *et al.* 2007), increasing the supply of models and inference procedures available for applied work.

The MNL framework has been used in the agricultural production economics literature to model acreage share choices (*e.g.*, Caswell and Zilberman 1985; Bewley *et al.* 1987; Lichtenberg 1989; Wu and Segerson 1995), crop decisions (Livingston *et al.* 2008) or land use decisions (*e.g.*, Lubowski *et al.* 2006). The acreage share models built within the MNL framework are mainly used for three reasons: (i) they ensure that the predicted share functions (strictly) lie in the interior of the zero-one interval, (ii) they are parsimonious in parameters and (iii) they are empirically tractable thanks to the so-called log-linear transformation. Nevertheless, the MNL framework was mainly employed for modeling plot level discrete decisions, the work of Wu and Segerson (1995) being a notable exception in this respect (see also Bewley *et al.* 1987). Furthermore, the MNL acreage share models were not integrated into economic production choice models and, as a result, were mainly used as convenient empirical functional forms for modeling acreage share choices.

The first objective of this article is to extend the pioneering works of Caswell and Zilberman (1985) and of Wu and Segerson (1995) by developing further the theoretical background of the MNL acreage share models. The (standard) MNL acreage share functional form considered in this article is defined as:

$$s_k(\boldsymbol{\pi},\mathbf{c},a) = \frac{\exp\left[a(\pi_k - c_k)\right]}{\sum_{m=1}^{K}\exp\left[a(\pi_m - c_m)\right]}, \quad k = 1,...,K \tag{1}$$

where K is the number of crops. In past studies using MNL acreage share models, the arguments of the exponential functions in (1) were defined as linear functions of crop choices determinants. It is shown in the present article that these reduced form functions can be replaced by the $a(\pi_k - c_k)$ terms, where $\pi_k - c_k$ is a well defined measure of crop k profitability and where a is a parameter with a simple interpretation.

Wu and Segerson (1995) defined model (1) by considering profit maximization at the farm level with land as an allocatable fixed input and used MNL functional forms for their empirical acreage share equations. But they did not provide the link between their theoretical and empirical models. It is shown in this article that the MNL acreage share model can be derived from a farm level profit maximization program where the (restricted) profit function is defined as the weighted sum of the crop gross margins (the weights being the acreage shares) minus an "implicit cost function" of the chosen acreage. This approach is hereafter called the "cost function approach".

Caswell and Zilberman (1985) derived model (1) by aggregation at the farm level of crop (discrete) choices at the plot level. This approach is hereafter called the "discrete choice approach". In the present article, this approach is first extended in a simple dynamic setting with acreage adjustment costs in order to address a problem not considered before: although crop decisions are made at the plot level, these decisions are not independent from each other.

The second objective of this article is to show that MNL acreage share models can be used to define simple multi-crop econometric models with land as an allocatable fixed input. These models are systems composed of yield supply, variable input demand and acreage share demand equations. The econometric models derived along these lines are fairly easy to implement in practice. Their simple structure is particularly useful in research projects involving a linkage between economic and agronomic models, *e.g.*, in order to infer environmental impacts of land use decisions.[1]

The third purpose of this article is to suggest generalizations of the standard MNL acreage share model (1). It is shown that Nested MNL acreage share functions can be derived following the "cost function approach". The other proposed generalization of the standard MNL acreage share model extends the "discrete choice approach" of Caswell and Zilberman (1985) in a dynamic setting by exploiting the flexibility of the MNL discrete choice models. Using Rust's (1987) framework, it is possible to build dynamic acreage choice models accounting for crop rotation effects, one of the major motives for crop diversification. Livingston *et al.* (2008) also use the dynamic MNL discrete choice framework, but their perspective is normative and defined at the plot level.

[1] These models are developed by the authors for that purpose.

The article is organized as follows. The assumptions necessary to build MNL acreage share models are discussed in the second section. The third and the fourth sections present the "cost function based approach" and the "discrete choice based approach" for defining standard MNL acreage share models. The fourth section also presents the standard "discrete choice" MNL acreage share model with partial adjustment costs. In the fifth section an application of the proposed multi-crop econometric models integrating the "cost function" and "discrete choice" (standard) MNL acreage share models is presented. It aims at illustrating the empirical relevance of the proposed models, at comparing them and at presenting their limits. In the sixth section two generalizations of the standard MNL acreage share models are presented to illustrate the potential of this framework as a basis for modeling acreage decisions. The last section provides concluding remarks and proposes directions for further research.

2. Main assumptions on the multi-crop production technology

The modeling frameworks used by agricultural economists to represent farmers' acreage decisions differ by their focus on one or two of the main motives for crop diversification: decreasing marginal return to crop acreages (or more generally scale and scope economies), (production or/and price) risk spreading, constraints associated to allocated quasi-fixed factors (other than land) or crop rotation effects. Multi-crop econometric models considering land as fixed but allocatable mostly focus on decreasing marginal returns to crop acreage (see, *e.g.*, Just *et al.* 1983; Chambers and Just 1989; Moore and Negri 1992) and on risk spreading (see, *e.g.*, Chavas and Holt 1990; Coyle 1992) as the motives for crop diversification. Crop rotation effects are more rarely considered in multi-crop econometric models, probably due to the complexity of dynamic choice modeling (see, *e.g.*, Ozarem and Miranowski 1994; Thomas 2003). Although they also consider other motives for crop diversification the distinctive feature of the (positive) mathematical programming ((P)MP) models is that they allow to consider constraints on acreage choices faced by farmers (Howitt 1995).

The MNL models are mainly built by considering the constraints on acreage choices as the farmers' motive for crop diversification. These constraints are agronomic constraints (impossible or "forbidden" rotations) and/or constraints associated with limiting quantities of quasi-fixed inputs (labor, machinery…). These constraints are represented by the acreage management cost function in the "cost function approach" and by adjustment costs in the "discrete choice approach". In this respect, this article contributes to the growing literature linking programming models and duality-based models (see, *e.g.*, Howitt 1995; Heckeleï and Wolff 2003).

The MNL acreage share models presented in this article share several assumptions. The first is the farmers' risk neutrality. This assumption is innocuous where risk issues can be neglected. Although it appears restrictive, it is imposed in models not considering risk spreading motives for crop diversification.

Besides this assumption on farmers' attitude toward risk, the derivation of MNL acreage share models is based on two main assumptions related to the production technology: (1) the crop marginal short run returns to land are assumed to be constant in the acreage levels and (2) variable input uses are assumed to not depend on quasi-fixed input quantities, at least "locally". What is meant by "locally" is defined in what follows. These assumptions are uncommon and deserve comments. Constant short run returns to acreages is used as a simplifying assumption in multi-crop econometric models considering risk spreading as the motive for crop diversification or in ((P)MP) models. This assumption is also imposed by Wu and Segerson (1995). The MNL models considered here can not be extended to accommodate marginal gross margins decreasing in acreages, at least as far as the usual representation of these scale effects is considered. However the "dynamic discrete choice" MNL acreage share model presented in the last section accounts for one the main source of decreasing short run returns to land, *i.e.* crop rotation effects. As the acreage allocated to a given crop increases, farmers need to allocate land with less favorable crop rotation effects. In this respect the "dynamic discrete choice" MNL acreage share model account for decreasing marginal gross margins according to step effects. Crop returns are constant for a given crop rotation. They decrease as the crop is produced on plots with less suitable crop rotation effects. The second assumption is admittedly restrictive as it implies that the variable input uses depend on the available quantities of labor and machinery only through the acreage choices. The available quantities of quasi-fixed factors determine the shape of the implicit cost function of the "cost function approach" and the adjustment costs of the "discrete choice approach". The MNL framework is well suited if it is more profitable for farmers to adapt their land allocation choices to their available quasi-fixed input quantities rather than to adapt their variable input uses at the crop level. The agricultural scientists and the extension agents consulted by the authors usually assert that farmers are more reluctant to change their cropping practices than their land allocation, at least in the short run and within standard rotation patterns. The independence assumption of the quasi-fixed input requirements with respect to variable input uses can only hold "locally" (and approximately), *i.e.* for variable input uses in the neighborhood of the current use levels. This limits the applicability of MNL acreage share models to short run decisions, *i.e.* decisions only involving moderate changes in variable input use levels or decisions not involving technological changes. Under these conditions the MNL acreage choice models can be interpreted as "local" approximations of the "true" choice process of the farmers.

According to the usual structural interpretation of the multicrop technology, the MNL framework also imposes non-jointness restrictions of the multicrop technology in variable inputs, in outputs and in acreages. Non-jointness in variable inputs and in outputs is commonly assumed while non-jointness in acreages is more debated (see, *e.g.*, Just *et al.* 1983; Chambers and Just 1989; Asunka and Shumway 1996). However discussing these assumptions especially makes sense if the considered models are employed for investigating the properties of multicrop technology or for investigating drastic changes of the economic environment. The MNL acreage share models can not be used for these purposes. Much more flexible representations of the production technology are required. The agricultural production economics literature provides numerous examples of models much more relevant in this context (see, *e.g.*, Just and Pope 2001).

3. Acreage decisions within MNL framework: the cost function approach

In this section, it is shown that MNL acreage shares can be derived from a profit maximization program defined at the farm level. The presented framework considers a riskless environment but it is easily extended to cases with price and/or production risk as far as farmers are assumed to be risk neutral.

The typical short run problem faced by a farmer is to allocate his land to K different crops according to the acreage shares s_k for $k = 1,...,K$ with $\sum_{k=1}^{K} s_k = 1$. Crop k output is sold at price p_k. The $L \times 1$ vector of variable input prices is denoted by $\mathbf{w} \equiv \{w_\ell\}$. These prices are assumed to be known by the farmers for simplicity.

Each crop production technology is represented by a yield function of the form:

$$y_k = f_k(\mathbf{x}_k) \quad (2)$$

where y_k is the yield of crop *k*, $\mathbf{x}_k$ is the quantity vector of variable input uses per unit of land of crop *k* and $f_k(.)$ is assumed to be nondecreasing and concave in $\mathbf{x}_k$. As is discussed below, $\mathbf{x}_k$ only include fertilizers, pesticides and seeds. The yield functions only depend on variable inputs and thus mostly represent the biological crop production process. Land and variable inputs such as fertilizers, seeds and pesticides are directly involved in the crop growth and development processes. The other quasi-fixed inputs (mainly machinery and labor) and the other variable inputs (mainly energy) are used for the variable input applications, for harvesting or for the soil preparation. Also the availability of quasi-fixed inputs mostly plays an indirect role in the biological crop production process. The main benefit of this framework is that the yield functions $f_k(.)$ are similar to the ones considered by agricultural scientists.

Farmers' short run profit or gross margin function per hectare of any crop $k = 1,...,K$ is defined by:

$$\pi_k(p_k,\mathbf{w}) = Max_{y_k,\mathbf{x}_k}\left[p_k y_k - \mathbf{x}_k'\mathbf{w}\right] \quad s.t. \quad y_k = f_k(\mathbf{x}_k),\, y_k \geq 0,\, \mathbf{x}_k \geq \mathbf{0}\,. \tag{3}$$

This model describes short run production choices. It considers variable input choices varying within the range defined by the cropping practices used at the time the decisions are made. Moderate changes in cropping practices, *e.g.* in fertilizer use or pesticide use levels, do not change the short run production technologies, *i.e.* the $f_k(.)$ functions, and only slightly modify the requirements for the quasi-fixed input services. Drastic changes in cropping practices involve long term choices: adoption of new cropping practices involving changes in the yield functions $f_k(.)$ and adaptation of the quasi-fixed input quantities.

The farmers' restricted profit function explicitly defines a trade-off between the crop gross margins $\pi_k(p_k,\mathbf{w})$ of the different crops on the one hand and the "implicit management cost" of the chosen allocation $C(\mathbf{s})$ on the other hand:

$$\Pi(\mathbf{s};\mathbf{p},\mathbf{w}) = \sum_{k=1}^{K} s_k \pi_k(p_k,\mathbf{w}) - C(\mathbf{s}) \tag{4}$$

where $\mathbf{p} \equiv \{p_k\}$ and $\mathbf{s} \equiv \{s_k\}$. The cost function $C(\mathbf{s})$ defines the motive for crop diversification. It "concentrates" the non-linear effects of **s** in the farmers' restricted profit function. It can be interpreted as a reduced form function smoothly approximating i) the unobserved variable costs associated with a given acreage (energy costs, …) and ii) the effects of binding constraints on acreage choices, *e.g.* agronomic constraints or constraints associated to limiting quantities of quasi-fixed inputs. Quasi-fixed inputs such as labor and machinery are limiting in the sense that their cost per unit of land devoted to a given crop is likely to increase due to work peak loads or due to machinery overuse, whether machinery is specific or not. Some crop rotations are impossible due to inconsistencies in planting and harvesting dates. Crop rotations may also be strongly unwarranted due to dramatic expected pest damages. These crop rotations are thus almost “forbidden” because their opportunity cost is very large in standard price ranges. These impossible and “forbidden” crop rotations determine the bounds imposed to acreage choices in (P)MP models. The implicit cost function $C(\mathbf{s})$ is assumed to be nondecreasing and quasi-convex in **s** to reflect the constraints due to the limiting quantities of quasi-fixed factors (other than land) and due to the implicit bounds imposed on the acreage choices due to impossible or “forbidden” crop rotations. Its definition implies that $C(\mathbf{s})$ can also be assumed to decreasing in the available quantities of quasi-fixed inputs (other than land).

Restricted profit functions, similar to the one defined in (4), are used in the PMP literature (Howitt 1995; Paris and Howitt 1998).[2] Heckeleï and Wolff (2003) also propose to use this form of restricted profit function to define multi-crop econometric models with land as an allocatable fixed factor. The main differences between the cost function used here, $C(\mathbf{s})$, and the ones used in the PMP literature are that i) $C(\mathbf{s})$ includes the effects of all binding constraints on acreage choices and ii) $C(\mathbf{s})$ is defined with (cross-entropy) measures of **s** whereas the PMP implicit cost functions are usually quadratic in **s**. Such implicit cost functions are also considered in dynamic models to account for adjustment costs, see *e.g.* Oude Lansink and Stefanou (2001).

The above discussion highlights the main features and limitations of the basic assumption set required to derive the standard MNL acreage share model from a restricted profit function. The main feature of this required assumption set is that the farmers short run production choices can be defined as the results of two optimization programs. First, farmers choose the optimal objective yield and input uses for each crop by solving the programs in (3). Second,

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[2] In most PMP applications, the cost function is designed to measure the total variable cost of **s**. In this article, the observed variable input costs are part of the gross margins.

they choose the optimal acreages by maximizing $\Pi(\mathbf{s};\mathbf{p},\mathbf{w})$ in $\mathbf{s}$ subject to the land use constraint $\sum_{k=1}^{K} s_k = 1$.

Building on the work of Anderson *et al.* (1992) it can be shown that the maximization of $\Pi(\mathbf{s};\mathbf{p},\mathbf{w})$ in $\mathbf{s}$ subject to the land use constraint $\sum_{k=1}^{K} s_k = 1$ leads to acreage share functions with the MNL functional form defined by (1) if the cost function $C(\mathbf{s})$ has the following form:[3]

$$C(\mathbf{s}) = A + \sum_{k=1}^{K} s_k c_k + a^{-1} \sum_{k=1}^{K} s_k \ln s_k \,. \tag{5a}$$

The term A is an unidentifiable fixed cost. The c_k parameters are fixed costs (in the short run) per unit of land devoted to the crops k. The term $\sum_{k=1}^{K} s_k \ln s_k$ is the opposite of the entropy function of the acreage share vector $\mathbf{s}$. Given that the acreage shares strictly lie between 0 and 1 and sum to 1 this term is negative. It is also strictly increasing and strictly convex in $\mathbf{s}$. The term $\sum_{k=1}^{K} s_k \ln s_k$ is minimal at $s_k = K^{-1}$ for $k = 1,...,K$ implying that A can be chosen to ensure that the cost function is positive. This implicit management cost function has a fairly simple interpretation with an alternative but equivalent specification. Define the parameters $d_k = \exp(-ac_k) \Big/ \left[\sum_{m=1}^{K} \exp(-ac_m) \right]$ and $A^d = A - a^{-1} \ln \left[\sum_{m=1}^{K} \exp(-ac_m) \right]$ for $k = 1,...,K$. The implicit cost function can then be defined in the alternative functional form:

$$C(\mathbf{s}) = A^d + a^{-1} \sum_{m=1}^{K} s_k (\ln s_k - \ln d_k) \,. \tag{5b}$$

The term $\sum_{k=1}^{K} s_k (\ln s_k - \ln d_k)$ is the opposite of the cross-entropy function of the acreage share vector. Its minimum is achieved at $s_k = d_k$ for $k = 1,...,K$. The vector of parameters $\mathbf{d} \equiv \{d_k\}$ defines the acreage share vector for which the implicit management cost is minimum, *i.e.* the most suitable acreage for the farm according to the implicit management costs. These costs increase with the difference between $\mathbf{s}$ and $\mathbf{d}$ according to the distance defined by the opposite of the cross-entropy function. In the implicit cost function, the a parameter is assumed to be strictly positive. It defines the relative "weight" of the management costs in the restricted profit function. It can be shown that the farmer only grows

[3] The formal proof is provided in Appendix 1.

the most profitable crop if a goes to infinity and he chooses the minimum cost acreage **d** if a goes to 0. The parameter vector **d** (or $\mathbf{c} \equiv \{c_k\}$) and the parameter a depend on the available quasi-fixed factors quantities and the previous acreage choices.

Along with this simple interpretation, the MNL acreage share models have two other interesting properties. First, the congruent indirect profit function $\Pi^*(\mathbf{p},\mathbf{w})$ and Lagrange multiplier associated to the land constraint $\lambda^*(\mathbf{p},\mathbf{w})$ have simple closed-form solutions which depend on the well-known log-sum term:

$$\Pi^*(\mathbf{p},\mathbf{w}) \equiv a^{-1}\ln\left[\sum_{k=1}^{K}\exp\left[a\left(\pi_k(p_k,\mathbf{w})-c_k\right)\right]\right]-A \quad \text{and} \quad \lambda^*(\mathbf{p},\mathbf{w}) \equiv a\left[\Pi^*(\mathbf{p},\mathbf{w})+A\right]. \tag{6}$$

Second, using the so-called log-linear transformation, the MNL acreage share equations can be defined as: [4]

$$\ln s_k - \ln s_K = a\left[\left(\pi_k(p_k,\mathbf{w})-\pi_K(p_K,\mathbf{w})\right)-(c_k-c_K)\right]+u_k \tag{7a}$$

or:

$$\ln s_k - \ln s_K = a\left(\pi_k(p_k,\mathbf{w})-\pi_K(p_K,\mathbf{w})\right)+\ln(d_k/d_K)+u_k \tag{7b}$$

for $k=1,...,K-1$ and where the u_k are error terms. This specification of the acreage choice system of equations uses crop K as the reference crop. It is linear in the crop profit function and, as a result, can easily be incorporated into multi-crop econometric models including yield supply and input demand functions as shown by the illustrative applications.

4. Acreage decisions within MNL framework: the discrete choice approach

MNL acreage shares can also be defined as the result of plot by plot discrete decisions. The MNL acreage share model presented in this section is based on two main points: the aggregation of choices made at the plot level, along the lines of Caswell and Zilberman (1985), and the logic of partial adjustment of acreage choices. As will be discussed below, the partial adjustment framework is employed to account for adjustment costs and for constraints on acreage choices, *i.e.* to account for the fact that the crop choices made at the plot level are not independent from each other.

[4] Crop K is chosen as the reference crop without any loss of generality.

The farmer is assumed to own P ($n=1,...,P$) plots of equal size (for simplicity) and to decide which of the K crops to grow on each plot. It is assumed at this point that farmers' decisions depend only on the expected gross margins $\pi_{kn}(p_k,\mathbf{w})$ and on the implicit fixed "management" costs c_{kn} for crop k on plot n. Under the assumption of farmers' risk neutrality, these expected gross margins can formally be defined as in the preceding section. The farmer's expected short run profit of growing crop k on plot n is given by:

$$\pi_{kn}(p_k,\mathbf{w})-c_{kn}=\pi_k(p_k,\mathbf{w})-c_k+e_{kn} \tag{8}$$

for $n=1,...,P$ and $k=1,...,K$. The term e_{kn} is known to the farmer but is random from the econometrician's point of view. Its expectation is normalized to be null. The plots are assumed to be sufficiently homogenous for considering the expected profit of growing crop k to be constant across plots, it is then given by $\pi_k(p_k,\mathbf{w})-c_k$, and for assuming that e_{kn} terms are identically and independently distributed across plots and crops. The probability (as it is perceived by the econometrician) that the farmer chooses crop k for plot n has a standard multinomial Logit form:

$$P_{kn}(\mathbf{p},\mathbf{w})=\frac{\exp\left[\sigma^{-1}\left(\pi_k(p_k,\mathbf{w})-c_k\right)\right]}{\sum_{m=1}^{K}\exp\left[\sigma^{-1}\left(\pi_k(p_m,\mathbf{w})-c_m\right)\right]}=P_k(\mathbf{p},\mathbf{w}) \quad , \tag{9}$$

if the $\sigma^{-1}e_{kn}+e$ terms have a standard Extreme Value distribution, where e is the Euler constant, and σ is a scale parameter of the variance of the e_{kn} terms. The assumptions stating that σ does not depend on $(\mathbf{p},\mathbf{w})$ and that the e_{kn} terms are identically and independently distributed across plots and crops are consistent with the assumption that e_{kn} is part of c_{kn} , *i.e.* of the implicit cost of growing crop k on plot n. According to this interpretation the e_{kn} terms represent the effects of the plots' characteristics (topography, spatial distribution, ...). The homoskedasticity assumption related to the e_{kn} error terms can be relaxed, *e.g.*, to account for heterogeneity across farms of the expected gross margins $\pi_{kn}(p_k,\mathbf{w})$ due to differences in expected input uses or in expected yields. This would however result in more involved econometric models, at least in some cases.

If farmer's choices of crops were independent across plots, the expected (as it is perceived by the econometrician) share of plots allocated to crop k would be given by $P_k(\mathbf{p},\mathbf{w})$. Indeed the

$P_k(\mathbf{p},\mathbf{w})$ terms define the "ideal" choice of the farmer, *i.e.* the acreage shares the farmer would choose if he was not constrained in his acreage choices. In this sense the $P_k(\mathbf{p},\mathbf{w})$ terms describe a long term (*i.e.* with optimal fixed factors quantities) choice of acreage shares. According to this logic and assuming that the farm is close to an equilibrium path, the farmer's dynamic optimal choice of acreage shares can be approximated by a simple partial adjustment model (Treadway 1971; Considine and Mount 1984). A similar logic was used by Oude Lansink and Stefanou (2001). Denoting by s_k the share of land devoted to crop *k* and by $s_{k,(-1)}$ its counterpart for the preceding year, the resulting partial adjustment model is given by:

$$\ln s_k - \ln s_{k,(-1)} = r\left(\ln P_k(\mathbf{p},\mathbf{w}) - \ln s_{k,(-1)}\right) + \varepsilon_k \text{, for } k = 1,\dots,K \quad (10)$$

where ε_{kt} is an error term including the approximation error due to the use of the simple adjustment model as well as the error term due to the use of $P_k(\mathbf{p},\mathbf{w})$ in place of the true long term choice of the farmer. Parameter *r* is the coefficient measuring the "friction" effects due to adjustment costs. It lies between 0 and 1 and can be defined in the empirical model as a function of available quasi-fixed factor quantities.

Differentiation of equation (10) for crop *k* and the reference crop *K* leads to the following equation:

$$\ln\left(s_k s_K^{-1}\right) = r\sigma^{-1}\left(\pi_k(p_k,\mathbf{w}) - \pi_K(p_K,\mathbf{w})\right) - r\sigma^{-1}\left(c_k - c_K\right) + \left(1-r\right)\ln\left(s_{k,(-1)} s_{K,(-1)}^{-1}\right) + \varepsilon_k - \varepsilon_K \quad (11)$$

for $k = 1,\dots,K-1$. Equation (11) is close to equation (7b) as could be expected: the "cost function approach" and the "discrete choice approach" with partial adjustment both rely on implicit management costs of the acreages. According to the assumption that farms are close to an equilibrium path, the lagged acreage shares in equation (11) are acreage shares with low management costs. In this sense, the lagged acreage choice vector $\mathbf{s}_{(-1)} \equiv \left\{s_{k,(-1)}\right\}$ is close to the reference acreage $\mathbf{d}$. Parameter *r* defines the weight of the "target" or ideal acreage choices relative to the adjustment costs. The partial adjustment MNL acreage share model can also be used within a production choice system of equations. An empirical illustration is presented in the next section.

5. Empirical illustration

This section presents two simple applications of the modeling frameworks presented in the second and third sections, *i.e.* the "cost function" MNL acreage share model and the partial adjustment "discrete choice" MNL acreage share model.

Data

The data base is a rotating panel data sample (3 years *per* farm on average) of 5986 observations of French grain crop producers over the years 1989 to 2006, obtained from the Farm Accountancy Data Network (FADN). It provides detailed information on crop production for each farm i and year t: acreage s_{kit}, yield y_{kit} and price at the farm gate p_{kit} for each crop k. The FADN only provides aggregate data on variable input (pesticides, fertilizers, seeds and energy) expenditures whereas input price indices are made available at the regional level. Variable input quantities are aggregated into a single variable input for simplicity. X_{it} denotes the per hectare quantity of input purchased by farm i during year t and w_{it} denotes the corresponding price index. Total land area is used to control for scale effects in the presented empirical models. Acreage choices of three crops are considered: wheat, other cereals (mainly barley and corn) and, oilseeds (mainly rapeseed) and protein crops (mainly peas). Root crops (sugar beets and potatoes) acreages were considered exogenous due to the sugar beet quota system implemented in the UE and because most of the potato acreages are defined by contracts. Fodder crop acreage (mainly silage corn) was also considered as exogenous due to feeding constraints.

Multi-crop econometric models

The quadratic functional form is chosen for the yield functions for three reasons. First its congruent dual functions have simple functional forms. Second, the quadratic production function can be parameterized in a form which is fairly easy to interpret by agricultural scientists or extension agents. Third, the resulting yield supply, input demand and (indirect) gross margin functions can be generalized to account for farms and farmers unobserved heterogeneity and for production stochastic events in a "natural" way, *i.e.* by introducing additive random terms with simple interpretations. Pope and Just (2003) used this parameterization of the quadratic production function for this reason, albeit in a different context. Yield functions are defined as:

$$y_{kit} = f_{kit}(x_{kit}) = \alpha_{kit}(\mathbf{a}_k^{\alpha}) - \frac{1}{2}\gamma_{kit}^{-1}(\mathbf{a}_k^{\gamma})\left[\beta_{kit}(\mathbf{a}_k^{\beta}) - x_{kit}\right]^2 \tag{12a}$$

with:

$$\alpha_{kit}(\mathbf{a}_k^{\alpha}) \equiv \alpha(\mathbf{z}_{it};\mathbf{a}_k^{\alpha}) + \varepsilon_{kit}^{y}, \quad \beta_{kit}(\mathbf{a}_k^{\beta}) \equiv \beta(\mathbf{z}_{it};\mathbf{a}_k^{\beta}) + \varepsilon_{kit}^{x} \text{ and } \gamma_{kit}(\mathbf{a}_k^{\gamma}) \equiv \gamma(\mathbf{z}_{it};\mathbf{a}_k^{\gamma}) \quad (12b)$$

for $k=1,...,K$ where x_{kit} is the quantity of variable input used per hectare devoted to crop k by farmer i at t.[5] The terms $\mathbf{a}_k \equiv (\mathbf{a}_k^{\alpha},\mathbf{a}_k^{\beta},\mathbf{a}_k^{\gamma})$ are parameter vectors to be estimated, the vector $\mathbf{z}_{it}$ contain variables used to control for farm heterogeneity, variations in production levels over time and technological changes, and $\alpha(.)$, $\beta(.)$ and $\gamma(.)$ are known functions. The ε_{kit}^{y} and ε_{kit}^{x} terms are random terms representing farms unobserved heterogeneity and the effects on production of stochastic events such as climatic conditions or pest infestations. In this primal framework, the $\alpha_{kit}(\mathbf{a}_k^{\alpha})$ and $\beta_{kit}(\mathbf{a}_k^{\beta})$ terms have direct interpretations: $\beta_{kit}(\mathbf{a}_k^{\beta})$ is the variable input quantity required to achieve the maximum yield $\alpha_{kit}(\mathbf{a}_k^{\alpha})$. Both terms need to be positive. The $\gamma_{kit}(\mathbf{a}_k^{\gamma})$ term determines the curvature of the yield function and, as a result, determines the magnitude of the price effects. They need to be positive for the yield function to be concave. These parameters have direct "agronomic" interpretations allowing the results to be "checked" with agricultural scientists and extension agents.

Farmers' acreage choices are based on the expected crop gross margins. Prices are assumed to be known by farmers in this illustrative application. Maximization in x_k of the expectation gross margin $p_{kit}f_{kit}(x_k) - w_{it}x_k$ for each crop k leads to gross margin functions of the form:

$$\pi_{kit}(\mathbf{a}_k) + e_{kit}^{\pi} \equiv \left[p_{kit}\alpha_{kit}(\mathbf{a}_k^{y}) - w_{it}\beta_{kit}(\mathbf{a}_k^{\beta}) + p_{kit}\frac{1}{2}\gamma_{kit}(\mathbf{a}_k^{\gamma})(w_{it}^2/p_{kit}) \right] + e_{kit}^{\pi} \quad (13)$$

where $e_{kit}^{\pi} \equiv p_{kit}(e_{kit}^{y} - e_{Kit}^{y}) - w_{it}(e_{kit}^{x} - e_{Kit}^{x})$. e_{kit}^{y} and e_{kit}^{x} are farmers' expectations of the ε_{kit}^{y} and ε_{kit}^{x} terms at the time they choose their acreages.[6] The nice feature of the quadratic yield functions in this framework is that their congruent yield supply, input demand and gross margin functions have additive error terms with simple interpretations. The e_{kit}^{y} and e_{kit}^{x} error terms account for the unobserved (not controlled by $\mathbf{z}_{it}$) heterogeneity across farms and time in the yield functions. These terms are known to the farmers when they choose their acreages.

[5] Extension of the yield function to the multiple input case is straightforward.

[6] The (per hectare of grain crop) compensatory payments provided by the CAP are added to this gross margin functions in the estimated multi-crop econometric models.

Their mean is normalized to 0. The $\varepsilon_{kit}^{y}-e_{kit}^{y}$ and $\varepsilon_{kit}^{x}-e_{kit}^{x}$ terms account for the stochastic events affecting production levels. These terms are unknown to the farmers when they choose their acreages, but there are known to them when they decide their variable input quantities. These error terms are unknown to the econometrician and their expectation is normalized to 0.

The considered multi-crop econometric models are defined by equation systems composed of K yield supply functions:

$$y_{kit}=\alpha_{kit}(\mathbf{a}_k^{y})-\frac{1}{2}\gamma_{kit}(\mathbf{a}_k^{\gamma})(w_{it}/p_{kit})^2+\varepsilon_{kit}^{y}\text{, } k=1,...,K\text{,} \quad (14)$$

an aggregate input demand equation:[7]

$$X_{it}=\sum_{k=1}^{K+M}s_{kit}\left(\beta_{kit}(\mathbf{a}_k^{\beta})-\gamma_{kit}(\mathbf{a}_k^{\gamma})(w_{it}/p_{kit})\right)+\left(\sum_{k=1}^{K}s_{kit}\varepsilon_{kit}^{x}+\varepsilon_{it}^{X}\right) \quad (15)$$

and $K-1$ acreage equations. These equations are defined, for the "cost function" MNL acreage share model, by:

$$\ln(s_{kit}s_{Kit}^{-1})=a_{it}(\mathbf{b})\left(\pi_{kit}(\mathbf{a}_k)-\pi_{Kit}(\mathbf{a}_K)-c_{kit}(\mathbf{g}_k)\right)+\left(e_{kit}^{\pi}+e_{kit}^{c}\right)\text{, } k=1,...,K-1\text{,} \quad (16)$$

and, for the "discrete choice" MNL acreage share model with partial adjustment, by: .

$$\ln\left(s_{kit}s_{Kit}^{-1}\right)=r\sigma^{-1}\left(\pi_{kit}(\mathbf{a}_k)-\pi_{Kit}(\mathbf{a}_K)-c_k\right)+\left(1-r\right)\ln\left(s_{ki,t-1}s_{Ki,t-1}^{-1}\right)+\left(e_{kit}^{\pi}+e_{kit}^{c}\right)$$
$$k=1,...,K-1 \quad (17)$$

The terms $\mathbf{b}$, $\mathbf{g}_k$, c_k and r are parameters to be estimated. The fixed cost per hectare of crop k is defined by $c_{kit}(\mathbf{g}_k)\equiv c(\mathbf{z}_{it};\mathbf{g}_k)+e_{kit}^{c}$ in equations (16) and by $c_k+e_{kit}^{c}$ in equations (17). The normalization constraints $c_{Kit}\equiv 0$ and $c_{Kit}(\mathbf{g}_K)\equiv 0$ reflects the fact that only the differences in the fixed cost terms $c_{kit}-c_{Kit}$ and $c_{kit}(\mathbf{g}_k)-c_{Kit}(\mathbf{g}_K)$ can be recovered for $k=1,...,K-1$. The error terms e_{kit}^{c} are known to the farmers but unknown to the econometrician. Their expectation is normalized to 0. The error term added to the input demand equation (14), ε_{it}^{X}, account for measurement errors due to stock variations.

[7] In the application the input uses for root crops and fodder crops are added in the input use equations ($k=K+1,...,M$). The corresponding input uses are defined as linear functions of the $\mathbf{z}_{it}$ variable vector defined below (cubic time trends, quadratic effects of the production potential index ...).

The $\mathbf{z}_{it}$ variable vector contains control variables. Quadratic time trends were introduced in the yield functions to account for (disembodied) technical changes. A "production potential index", $q_{it} \equiv (y_{1i,t-1} - y_{1i,t-1}^{Med}) / (y_{1i,t-1}^{Max} - y_{1i,t-1}^{Min})$, is created to control for farm heterogeneity. $y_{1i,t-1}^{Med}$, $y_{1i,t-1}^{Max}$ and $y_{1i,t-1}^{Min}$ denote, respectively, the median, 99% quantile and 1% quantile of the yield of wheat in the sample in year $t-1$. It is based on wheat yields due to the specialization of the sampled farms, and it is defined on a year by year basis to control for year specific conditions. Quadratic effects of q_{it} are introduced in the parameters of the yield functions. The specified effects of q_{it} can be interpreted as control functions of the farms' heterogeneity. This control function approach is analogous to Chamberlain's (1982) Π matrix approach and to Mundlak's (1978) device for controlling for the so-called individual fixed effects in panel data econometrics. While this index mostly accounts for persistent production conditions, farmers' choices and yields also depend on crop rotation effects. The lagged acreage shares of root crops are introduced in the cereal yield functions to account for the beneficial effects of the induced crop rotations.

Estimation issues

The control variable vector $\mathbf{z}_{it}$ and the price variables are exogenous with respect to the error terms of the econometric models. The $\alpha_{kit}(\mathbf{a}_k^y)$, $\gamma_{kit}(\mathbf{a}_k^\gamma)$, $\beta_{kit}(\mathbf{a}_k^\beta)$ and $c_{kit}(\mathbf{g}_k)$ terms are defined as linear functions of $\mathbf{z}_{it}$ and are linear in their respective parameters. The $a_{it}(\mathbf{b})$ term is defined as the exponential of a linear function of $\mathbf{z}_{it}$ and $\mathbf{b}$. The acreage shares are potentially endogenous in the input demand equation. The ε_{kit}^x error terms contain the heterogeneity effects e_{kit}^x which partly determine the acreage choices. In this illustration it is assumed that the heterogeneity control ensured by q_{it} in the crop input demand functions is sufficient to neglect the effects of the e_{kit}^x terms (the q_{it} index is defined for that purpose). Albeit it is standard (see, *e.g.*, Hornbaker *et al.* 1989), this assumption is admittedly restrictive. Note however that this estimating equation is only needed for identifying $\mathbf{a}_K^\beta$. The acreage share equations identify the whole set of parameters $\mathbf{a}^\beta$ excluded but they also identify $\mathbf{a}_k^\beta - \mathbf{a}_K^\beta$ for $k = 1,...,K-1$.

The parameter estimators are constructed within the Generalized Method of Moment (GMM) framework. [8]They are based on the orthogonality conditions defined by the vector of the cross product of the (composite) error term of each equation with each of their exogenous explanatory variables. The resulting GMM estimator is robust to heteroskedasticity of unknown form and does not exclude correlation of the error terms across equations.

Main results

Table 1 presents the estimates of yield supply, input demand and acreage shares function parameters for the "cost function" and "discrete choice" models. Table 2 presents the average price elasticities of the crop supply, input demand and acreage share functions.

Both models yield similar results with respect to the input demand and yield supply function parameters. The fit of the models to these micro-level data is correct. The R^2 criteria lie between .32 and .43. Estimates of the maximum yield and input requirements for maximum yield, *i.e.* the $\alpha_{kit}(\mathbf{a}_k^y)$ and $\beta_{kit}(\mathbf{a}_k^\beta)$ terms, are in the ranges expected by the agricultural scientists and extension agents the authors have consulted. As expected, the production potential index has positive effects on $\alpha_{kit}(\mathbf{a}_k^y)$ and $\beta_{kit}(\mathbf{a}_k^\beta)$ terms. Past acreages of root crops have a positive effect on wheat yield and a negative effect on the demand of wheat variable inputs. These effects are consistent with the known beneficial effects of root crops at the beginning of the crop rotation sequence. Estimated average price elasticities of the yield supply and crop input demands are reported in table 2. They lie in standard ranges, albeit the price responsiveness of the "other cereals" functions is surprisingly low. This may be explained by the inclusion of fairly different crops in this aggregate. Nevertheless these results demonstrate that both multi-crop models provide satisfactory econometric modeling frameworks: they yield sensible estimated price effects and expected heterogeneity control variable effects.

[8] The econometric models are not standard Seemingly Unrelated Regression systems despite that they are composed of regression equations only. The acreage shares are the dependent variables of the acreage equations whereas they are independent variables in the input demand equation.

Table 1: Estimates of the Yield, Input Demand and Acreage Shares Equations, 1989-2006

	"Cost function" model			*"Discrete choice" model*		
Explanatory Variable	Wheat	Other cereals	Oilseeds protein crops	Wheat	Other cereals	Oilseeds protein crops
Yield supply						
Price effects (γ)	1.89^{***}	1.34^{***}	1.71^{***}	1.56^{***}	0.75^{***}	1.83^{***}
Production index	0.48*	0.19	3.06^{***}	0.57^{*}	-0.64^{**}	2.83^{***}
Average potential yield (α)	8.69^{***}	8.28^{***}	6.74^{***}	8.56^{***}	8.02^{***}	6.79^{***}
Constant	7.89^{***}	8.36^{***}	5.86^{***}	7.98^{***}	8.08^{***}	6.02^{***}
Trend	0.12^{***}	0.04^{***}	0.11^{***}	0.09^{***}	0.10^{***}	0.09^{***}
Trend square	$-3\ 10^{-3***}$	$2\ 10^{-3**}$	$-5\ 10^{-3***}$	$-2\ 10^{-3***}$	$-2\ 10^{-3***}$	$-4\ 10^{-3***}$
Production index	2.68^{***}	2.14^{***}	2.75^{***}	2.77^{***}	1.94^{***}	2.58^{***}
Root crop acreage	2.42^{***}	-	-	2.22^{***}	-	-
R-square	*0.42*	*0.34*	*0.32*	*0.42*	*0.34*	*0.32*
Input demand						
Average optimal input use (β)	5.18^{***}	5.51^{***}	5.40^{***}	4.80^{***}	5.01^{***}	5.25^{***}
Constant	5.53^{***}	5.67^{***}	6.13^{***}	5.63^{***}	4.98^{***}	5.71^{***}
Production index	1.84^{***}	-0.46	3.29^{***}	1.34^{***}	0.10	3.16^{***}
Root crops		10.13^{***}			11.61^{***}	
Fodder crops		1.44			1.75	
R-square		*0.42*			*0.44*	
Acreage shares						
Fixed costs (c)	-	-2.90^{***}	-1.50^{***}	-	-1.17^{***}	-1.28^{***}
Production index	-	-2.88^{***}	0.58	-	-	-
Root crops	-	-23.16^{**}	-40.05^{**}	-	-	-
Fodder crops	-	3.84^{***}	5.98^{***}			
Cost weight (a)		0.14^{***}			-	
Root crops		-2.85^{***}			-	
Friction parameter (r)		-			0.25^{***}	
Scale parameter (σ)		-			0.19^{***}	
R-square	-	*0.11*	*0.19*	-	*0.59*	*0.51*

Note: (*), (**) and (***) denote parameter estimates statistically different from 0 at, respectively, 10%, 5% and $\leq$1% confidence levels.

Table 2: Estimated average price elasticities of yield supply, input demand and acreage shares

	"Cost function" econometric model				*"Discrete choice" econometric model*			
	Price				**Price**			
	Wheat	Other cereals	Oilseeds protein crops	Input	Wheat	Other cereals	Oilseeds protein crops	Input
Yield supply functions								
Wheat	0.178	-	-	-0.178	0.143	-	-	-0.143
Other cereals	-	0.156	-	-0.156	-	0.090	-	-0.090
Oilseeds, protein crops	-	-	0.234	-0.234	-	-	0.248	-0.248
Input demand functions								
Wheat	0.427	-	-	-0.427	0.353	-	-	-0.353
Other cereals	-	0.275	-	-0.275	-	0.158	-	-0.158
Oilseeds, protein crops	-	-	0.419	-0.419	-	-	0.460	-0.460
Acreage share functions, short-run								
Wheat	0.569	-0.269	-0.228	-	0.228	-0.107	-0.090	-
Other cereals	-0.479	0.702	-0.228	-	-0.199	0.285	-0.090	-
Oilseeds, protein crops	-0.479	-0.269	0.617	-	-0.199	-0.107	0.253	-
Acreage share functions, long-run								
Wheat	-	-	-	-	0.978	-0.461	-0.388	-
Other cereals	-	-	-	-	-0.856	1.221	-0.388	-
Oilseeds, protein crops	-	-	-	-	-0.856	-0.461	1.087	-

Estimates of the acreage equations lead to more contrasted conclusions depending on considered acreage share models. As expected, the "cost function" acreage share model shows that farms with large root crop acreages devote also more land to cereals rather than to oilseeds and protein crops. The estimated $a_{it}(\mathbf{b})$ terms imply that the (expected) crop gross margin variations account for about 14% of variations of the differences in log acreage shares. As expected the estimated $a_{it}(\mathbf{b})$ terms are decreasing in past root crop acreages, indicating that these crops offer much flexibility for subsequent crop choices. The own price elasticities of the crop acreage shares (see Appendix 3) presented in table 2 range from .57 to .70. These estimated average elasticities are close to each other because their values mostly depend on the estimated values of the single term $a_{it}(\mathbf{b})$. The Nested MNL acreage share model which is presented in the next section offers much flexibility in this respect. These results globally indicate that the "cost function" MNL acreage share model provide sensible results with respect to price effects and heterogeneity control variables effects. Nevertheless, the low R^2 criteria (.11 and .19) for these acreage equations call for improvement of the econometric "cost function" MNL acreage share model with respect to the use of extra variables to better control for heterogeneity. This lack of fit may also be due to the CAP instruments implemented in the period covered by our data. Price supports to grain crops sharply declined in the EU during the nineties. But this decrease in price support has been compensated by direct payments (which are incorporated in the empirical models). These direct payments were defined for each grain crop for compensating producers' gross margins at the *département* level (France is divided into 95 *départements*). As a result grain crop acreages have been "frozen" due to the implied negative correlation between the direct payments and crop gross margins.

The "discrete choice" acreage share model with partial adjustment model has a much better fit to the data. This is not surprising for a model basically predicting acreages in year t by acreages in year $t-1$. However, the estimated value of the "friction" parameter r is equal to .25 showing that acreages respond to short run economic incentives despite significant adjustment costs. This model allows to compute price elasticities of the crop acreage shares in the long run, *i.e.* without adjustment constraints, and in the short run, *i.e.* with limited adjustment possibilities. The own price average long run elasticities of the crop acreage shares are close to 1. The corresponding short run average elasticities are close to .25. As expected, the estimated price elasticities derived from the "cost function" model lie between the short run and long run elasticities derived from the "discrete choice" partial adjustment model.

It is also interesting to note that the *per* hectare "fixed costs" terms of both acreage share models tend to show that these models underestimate the oilseeds and protein crops acreages. A modified version of the model incorporating a crude measure of the beneficial effects of the oilseeds/protein crops-wheat rotation on future wheat gross margins provides a "correction" for this underestimation problem. This suggests that dynamic generalizations of the MNL acreage share models accounting for crop rotation effects may provide significant improvements for acreage choice modeling. The next section briefly presents the basic framework for building "dynamic discrete choice" MNL acreage share models.

6. Generalized MNL acreage share models

The main aim of this section is to present a brief overview of the possible generalizations of the MNL framework for acreage choice modeling. Other generalizations are possible. For example, the Nested MNL acreage share model derived using the "cost function approach" can also be derived by using the "discrete choice approach". Presenting these generalizations also allows to point out some drawbacks of the "standard" MNL acreage share models. Two generalizations of the standard multinomial Logit acreage share models are presented.

"Cost function approach": the Nested multinomial Logit model

The simplicity of the log-linear transformation used in equations (6) is mainly due to a specific feature of the MNL acreage shares. The ratio of the acreage shares of two different crops only depends on the payoffs of these crops. This "independence of the irrelevant crops" property also is a potential drawback of this simple acreage share model. The acreage share elasticities with respect to crop (expected) gross margins mainly depend on the single a parameter, *i.e.* the relative "weight" of the acreage management cost in the farmers' objective function. For example, the acreage share elasticities of crop k with respect to the price of crop ℓ are equal for $k = 1,...,C$ and $k \neq \ell$.

All crops are equivalently considered in terms of management costs in the MNL acreage share model. The "cost function" MNL acreage share framework can be generalized to account for similarities and differences in the management of the different crops, in the spirit of the PMP framework developed by Röhm and Dabbert (2003). If the K crops can be allocated to Q mutually exclusive nests according to their management costs, it is possible to define the corresponding "Nested MNL acreage share models" and their corresponding indirect profit and indirect restricted profit functions. The set of crops belonging to nest ℓ $(\ell = 1,...,Q)$ is denoted by $B(\ell)$, the share of land allocated to the crops of nest ℓ is denoted by $\bar{s}_\ell = \sum_{k\in B(\ell)} s_k$ and the share of crop k within its nest ℓ is denoted by $s_{k/\ell} = s_k / \bar{s}_\ell$. The price arguments of the gross margin functions are omitted to simply notations and the gross margin vector is denoted by $\boldsymbol{\pi} \equiv \{\pi_k\}$. Building on the work of Verboven (1996), it can be shown that the maximization in **s** of the restricted indirect profit function:

$$\Pi(\mathbf{s};\boldsymbol{\pi}) = \sum_{\ell=1}^{Q} \sum_{m\in B(\ell)} s_m(\pi_m - c_m) - a^{-1}\left[\sum_{\ell=1}^{Q} \bar{s}_\ell \ln \bar{s}_\ell + \sum_{\ell=1}^{Q} a\rho_\ell^{-1} \bar{s}_\ell \sum_{m\in B(\ell)} s_{m/\ell} \ln s_{m/\ell}\right]$$

subject to the total land allocation constraint leads to Nested MNL acreage share functions.[9],[10] If crop k belongs to nest q, we have:

$$s_k(\boldsymbol{\pi}) = \frac{\exp\left[\rho_q(\pi_k - c_k)\right]}{\sum_{m\in B(q)} \exp\left[\rho_q(\pi_m - c_m)\right]} \frac{\left[\sum_{m\in B(q)} \exp\left[\rho_q(\pi_m - c_m)\right]\right]^{a\rho_q^{-1}}}{\sum_{\ell=1}^{Q}\left[\sum_{m\in B(\ell)} \exp\left[\rho_\ell(\pi_m - c_m)\right]\right]^{a\rho_\ell^{-1}}} \tag{18}$$

In the restricted profit function a^{-1} is the weight parameter of the management cost function for the different nests while ρ_ℓ^{-1} is the weight parameter of the management cost function for the crops of nest ℓ. Note that the MNL restricted indirect profit functions and acreage share functions are special cases of their Nested MNL counterparts. The former is obtained from the latter with $\rho_\ell = a$ for $\ell = 1,...,Q$. The first right hand side term of equation (18) defines the share function of crop k within its nest, $s_{k/q}(\boldsymbol{\pi}_q)$ where $\boldsymbol{\pi}_q \equiv \{\pi_k,\ k\in B(q)\}$. The second right hand side term of equation (18) defines the share of total land allocated to nest q, $\bar{s}_q(\boldsymbol{\pi})$. Note that $\bar{s}_q(\boldsymbol{\pi})$ can also be written as:

$$\bar{s}_q(\boldsymbol{\pi}) = \frac{\exp\left[a\Pi_q^*(\boldsymbol{\pi}_q)\right]}{\sum_{\ell=1}^{Q}\exp\left[a\Pi_\ell^*(\boldsymbol{\pi}_\ell)\right]} \text{ with } \Pi_\ell^*(\boldsymbol{\pi}_\ell) \equiv \rho_\ell^{-1}\ln\left[\sum_{m\in B(\ell)}\exp\left[\rho_\ell(\pi_m - c_m)\right]\right] \tag{19}$$

[9] The formal proof is provided in Appendix 2.

[10] The indirect restricted profit function (as well as its congruent functions) defined with the “reference acreage share vector” is not given here but can be readily be derived.

The $\bar{s}_q(\boldsymbol{\pi})$ acreage share function is defined in a standard MNL form by using the indirect profit functions or inclusive value functions, $\Pi^*_\ell(\boldsymbol{\pi}_\ell)$, associated to the crops of the different nests ℓ. The Nested MNL framework is less tractable than the standard multinomial Logit framework since there is no simple counterpart to the log-transformation used with standard MNL models. However, in the particular cases where there is a single specific crop (an "outside" crop), the technique developed by Berry (1994) can be used to define empirically tractable estimating equations.[11]

"Discrete choice approach": MNL acreage share models accounting for crop rotation effects

The "discrete choice" MNL acreage share model can be generalized to account for the fact that farmers consider the expected crop rotation effects of their acreage choices. It is assumed for simplicity that production dynamics is of order 1 and that farmers only consider anticipations with respect to the next year with a discount factor *d*. The results presented in this section heavily rely on Rust's (1987) framework for discrete choice modeling. Let denote the profit of growing crop *k* on plot *n* where crop *m* was grown during the preceding year by:

$$\pi_{kn/m}(p_{kt},\mathbf{w}_t)-c_{kn/m}=\pi_{k/m}(p_{kt},\mathbf{w}_t)-c_{k/m}+e_{knt} \tag{20}$$

where the functional form of the $\pi_{kn/m}(.)$ functions is known and the e_{knt} terms are identically and independently distributed across crops, plots and time, and have the distribution defined in section 3. In year *t* the e_{knt} terms are known to the farmer but the $e_{kn,t+1}$ terms are not. It is however assumed that the farmer's perceived distribution of the $e_{kn,t+1}$ terms is also the distribution described in section 3. According to this model, if the prices $(\mathbf{p}_{t+1},\mathbf{w}_{t+1})$ are known to the farmer (and to the econometrician) then he knows that if he chooses crop *k* for plot *n* in *t*, the probability of his choosing crop ℓ in year $t+1$ on the same plot is given by:

$$P_{\ell/k}(\mathbf{p}_{t+1},\mathbf{w}_{t+1})=\frac{\exp\left[\sigma^{-1}\left(\pi_{\ell/k}(p_{\ell,t+1},\mathbf{w}_{t+1})-c_{\ell/k}\right)\right]}{\sum_{m=1}^{K}\exp\left[\sigma^{-1}\left(\pi_{m/k}(p_{m,t+1},\mathbf{w}_{t+1})-c_{m/k}\right)\right]}. \tag{21}$$

As a result his expected pay-off on plot *n* in year $t+1$ (as perceived in year *t*) is given by:

[11] The Nested MNL version of the econometric model provides interesting results which are similar to those presented in section 4.

$$E\left[\pi_{/k}(\mathbf{p}_{t+1},\mathbf{w}_{t+1})-c_{/k}\right]=\sigma\ln\left[\sum_{\ell=1}^{K}\exp\left[\sigma^{-1}\left(\pi_{\ell/k}(p_{\ell,t+1},\mathbf{w}_{t+1})-c_{\ell/k}\right)\right]\right] \tag{22}$$

i.e. the expected profit has the well-known log-sum form. Thus, in year t the (risk neutral) farmer (who has a perfect foresight on $t+1$ prices) chooses the crop on plot n according to the expected pay-offs given by:

$$\pi_{k/m}\left(p_{kt},\mathbf{w}_t\right)-c_{k/m}+e_{knt}+dE\left[\pi_{/k}(\mathbf{p}_{t+1},\mathbf{w}_{t+1})-c_{/k}\right] \tag{23}$$

From the econometrician point of view, the probability that the farmer chooses k has the standard MNL functional form:

$$P_{k/m}(\mathbf{p}_t,\mathbf{w}_t;\mathbf{p}_{t+1},\mathbf{w}_{t+1})=\frac{\exp\left[\sigma^{-1}\left(\pi_{k/m}(p_{kt},\mathbf{w}_t)-c_{k/m}+dE\left[\pi_{/k}(\mathbf{p}_{t+1},\mathbf{w}_{t+1})-c_{/k}\right]\right)\right]}{\sum_{q=1}^{K}\exp\left[\sigma^{-1}\left(\pi_{q/m}(p_{qt},\mathbf{w}_t)+dE\left[\pi_{/q}(\mathbf{p}_{t+1},\mathbf{w}_{t+1})-c_{/q}\right]\right)\right]}. \tag{24}$$

The closed form of the expected pay-off in year $t+1$ permits further generalizations. For example, uncertainty about prices in $t+1$ can be handled using integration of the expectation of the probability function (24) according to the assumed distribution of prices. Simulation methods that are now widely used can be employed for that purpose (see, *e.g.*, Train, 2003). These probability functions can also be used in the partial adjustment framework defined in the third section. Note however that the resulting empirical models remain close to the ones presented in this article only in the case where the crop rotations are observed. The resulting empirical models are more complicated where only acreages are observed. In this case the probability of choosing crop k at t is given by:

$$P_k(\mathbf{p}_t,\mathbf{w}_t;\mathbf{p}_{t+1},\mathbf{w}_{t+1})=\sum\nolimits_{m=1}^{K}s_{m,t-1}P_{k/m}(\mathbf{p}_t,\mathbf{w}_t;\mathbf{p}_{t+1},\mathbf{w}_{t+1}) \tag{25}$$

and the difference in the log-acreage shares does not provide any simplification. Nevertheless these crop choice probabilities may be simplified thanks to similarities of the rotation effects of certain crop sequences.

The agronomic constraints considered in the "cost function approach" differ from the crop rotation effects considered here. In the present framework, crop rotation effects generate inter-temporal trade-offs while the agronomic constraints considered in the "cost function approach" restrict acreage choices. In this respect, the "cost function approach" is only suitable where farmers' can be assumed to use restrictive rotation patterns whereas the "discrete choice approach" can be used as a modeling framework in broader situations.

7. Concluding remarks

Two approaches are presented that provide theoretical backgrounds for using MNL acreage share models: the "cost function" and the "discrete choice" approaches. The "discrete choice" based approach remains mainly empirical. This approach focuses on some of the farmers' decision parameters but it either ignores or only uses reduced form effects for the other farmers' decision parameters.[12] It exploits the flexibility of the MNL framework to focus on some determinants of the acreage choices, *e.g.* crop gross margins, acreage management costs or crop rotation effects. It ignores other determinants, *e.g.* risk spreading. The relevance of this choice depends on the context and needs to be empirically evaluated.

The "cost function" based approach appears to be more "structural" in the sense that it is based on profit functions. However, as it is the case for any simple theoretical model of production choices, the MNL acreage share models are to be used with caution. Just and Pope (2001) convincingly argue that any econometric model of farmers' choices necessarily contains reduced form effects because, among others, of the complexity of agricultural production processes, of the limitations of the usual data sets, of the complexity of the farmers' objective functions, ... The MNL acreage share models can be interpreted in two ways: either as a structural model relying on restrictive assumptions with respect to the underlying technology, or as a model approximating the "true" model. This second interpretation is preferred in this article. Introduction of appropriate control variables in empirical MNL acreage share models allows to define simple econometric models to be interpreted as local approximations of the "true" models and to be used to investigate the effects of moderate changes in the production context.

Both approaches accommodate generalizations of the standard MNL acreage share model. But the "discrete choice" MNL framework seems more flexible than its "cost function" counterpart. Accounting for crop rotation effects and for dynamic optimization by farmers appears to be a promising direction for further research as shown by the applications presented in this article as well as the results obtained by Livingston *et al.* (2008). These generalizations can benefit from the rapidly expanding literature on dynamic discrete choice econometric models.

Despite their limitations but thanks to their simple structure, the MNL acreage share models appear to be useful tools for investigating farmers' short run production decisions. They can

[12] Even though these reduced form effects can also be theoretically grounded.

be used to produce simple comparative statics results. They can also be used to build simple and reliable multi-crop econometric models as shown by the illustration presented in this article. Economists involved in multi-disciplinary research projects may also find it useful for defining production choice models which are likely to be preferred to the standard multi-crop dual models by non-economists thanks to the immediate interpretation of their parameters. The MNL acreage models also share another advantage with Mathematical Programming models: thanks to their simple structure they can easily be used for investigating the effects of new cropping practices on land allocation. Finally, these models can also be used by researchers as simple acreage choice models in more elaborated econometric models of production choice models.[13]

One of the main drawbacks of the MNL framework is that it rules out corner solutions in acreage shares. However, this certainly calls for original approaches for corner solution modeling.

[13] The MNL framework is the workhorse of the recent empirical industrial organization literature because it allows to define empirically tractable econometric demand functions which can be employed in various market equilibrium models (Ackerberg *et al.* 2007).

References

Ackerberg, D., Benkard, L., Berry, S., Pakes, A. (2007). Econometric tools for analyzing market outcomes. In Heckman, J.J., Leamer, E.E. (eds), *Handbook of Econometrics*, Vol. 6A, Amsterdam, Elsevier.

Anderson, S.P., de Palma, A., Thisse, J. (1992). *Discrete Choice Theory of Product Differentiation.* Cambridge, Mass, MIT Press.

Asunka, S., Shumway, C.R. (1996). Allocatable fixed inputs and jointness in agricultural production: more implications. *Agricultural and Resource Economics Review*, 25: 143-48.

Berry, S.T. (1994). Estimating discrete-choice models of product differentiation. *The RAND Journal of Economic*, 25(2): 242-262.

Bewley, R., Young, T., Colman, D. (1987). A system approach to modelling supply equations in agriculture. *Journal of Agricultural Economics*, 38(2): 151-166.

Caswell, M., Zilberman, D. (1985). The choice of irrigation technologies in California. *American Journal of Agricultural Economics*, 67(2): 224-234.

Chavas, J.-P., Holt, T. (1990). Acreage decisions under risk: the case of corn and soybeans. *American Journal of Agricultural Economics*, 72(3): 529-538.

Chambers, R., Just, R. (1989). Estimating multiouput technologies. *American Journal of Agricultural Economics*, 71(4): 980-95.

Considine, T.J., Mount, T.D. (1984). The use of linear logit models for dynamic input demand systems. *Review of Economics and Statistics*, 66(3): 434-443.

Coyle, B.T. (1992). Risk aversion and price risk in duality models of production: A linear mean-variance approach. *American Journal of Agricultural Economics*, 74(4): 849-859.

Heckeleï, T., Wolff, H. (2003). Estimation of constrained optimisation models for agricultural supply analysis based on generalised maximum entropy. *European Review of Agricultural Economics*, 30(1): 27-50.

Hornbaker, R.H., Dixon, B.L., Sonka, S.T. (1989). Estimating production activity costs for multioutput firms with a random coefficient regression model. *American Journal of Agricultural Economics*, 71(1): 167-177.

Howitt, E. (1995). Positive mathematical programming. *American Journal of Agricultural Economics*, 77(2): 329-342.

Just, R.E., Pope, R.D. (2001). The agricultural producer: theory and statistical measurement. In Gardner, B.L., Rausser, G.C. (eds), *Handbook of Agricultural Economics,* Vol. 1A. Amsterdam, Elsevier.

Just, R.E., Zilberman, D., Hochman, E. (1983). Estimation of multicrop production functions. *American Journal of Agricultural Economics*, 65(4): 770-780.

Lichtenberg, E. (1989). Land quality, irrigation development and cropping pattern in the northern high plains. *American Journal of Agricultural Economics*, 71(1): 187-94.

Livingston, M., Roberts, M.J., Rust, J. (2008). *Optimal Corn and Soybean Rotations*. Paper presented at the AAEA annual meeting, Orlando, Florida, 27-29 July.

Lubowski, R.N., Plantinga, A.J., Stavins, R.N. (2006). Land-use and carbon sinks: Econometric estimation of the carbon sequestration supply function. *Journal of Environmental Economics and Management*, 51(2): 135-152.

McFadden, D. (1974). Conditional logit analysis of quantitative choice behavior. In Zarembka, p. (ed), *Frontiers in Econometrics*, New-York, Academic Press.

Moore, M.R., Negri, D.H. (1992). A multicrop production model of irrigated agriculture, applied to Water Allocation Policy of the Bureau of Reclamation. *Journal of Agricultural and Resource Economics*, 17: 29-43.

Mundlak, Y. (1978). On the pooling of time series and cross section data. *Econometrica*, 46(1): 69-85.

Oude Lansink, A., Stefanou, S. (2001). Dynamic area allocation and economies of scale and scope. *Journal of Agricultural Economics*, 52(3): 38-52.

Ozarem, P.F., Miranowski, J.A. (1994). A dynamic model of acreage allocation with general and crop-specific soil capital. *American Journal of Agricultural Economics*, 76(3): 385-395.

Paris, Q., Howitt, R.E. (1998). An analysis of ill-posed production problems using maximum entropy. *American Journal of Agricultural Economic*, 80(1): 124-138.

Pope, R.D., Just, R.E. (2003). Distinguishing errors in measurement from errors in optimization. *American Journal of Agricultural Economics*, 85(2): 348-358.

Röhm, O., Dabbert, S. (2003). Integrating agri-environmental programs into regional production models: an extension of positive mathematical programming. *American Journal of Agricultural Economics*, 85(1): 254-265.

Rust, J. (1987). Optimal replacement of gmc bus engines: An empirical model of Harold Zurcher. *Econometrica*, 55(5): 999-1033.

Theil, H. (1969). A multinomial extension of the linear logit model. *International Economic Review*, 10(3): 251-259.

Thomas, A. (2003). A dynamic model of on-farm integrated nitrogen management. *European Review of Agricultural Economics*, 30(4): 439-460.

Train, K. (2003). *Discrete Choice Methods with Simulation*. Cambridge University Press.

Treadway, A.B. (1971). On the rational multivariate flexible accelerator. *Econometrica*, 39(5): 845-855.

Verboven, F. (1996). The nested logit model and representative consumer theory. *Economics Letters*, 50(1): 57-63.

Wu, J., Segerson, K. (1995). The impact of policies and land characteristics on potential groundwater pollution in Wisconsin. *American Journal of Agricultural Economics*, 77(4): 1033-1047.

Appendices

Appendix 1: Acreage shares in the standard MNL model

In the standard MNL case, the producer's program is provided by:

$$\underset{\mathbf{s}}{Max}\left[\sum_{k=1}^{K}\pi_k s_k - \left(A+\sum_{k=1}^{K}c_k s_k + a^{-1}\sum_{k=1}^{K}s_k \ln s_k\right)\right] \text{ subject to } \sum_{k=1}^{K}s_k = 1.$$

The Lagrangian function is defined by:

$$L(s,\lambda)=\sum_{k=1}^{K}\pi_k s_k - \left(A+\sum_{k=1}^{K}c_k s_k + a^{-1}\sum_{k=1}^{K}s_k \ln s_k\right)-\lambda\left(\sum_{k=1}^{K}s_k - 1\right).$$

It leads to the following first order conditions (FOCs):

(A1) $$\frac{\partial L}{\partial s_k}=\pi_k - c_k - a^{-1}\left(\ln s_k + 1\right)-\lambda = 0,\ k=1,...,K$$

(A2) $$\frac{\partial L}{\partial \lambda}=\sum_{m=1}^{K}s_m - 1 = 0,\ m=1,...,K$$

Equation (A1) leads to:

$$\pi_k - c_k - a^{-1}\ln s_k - a^{-1} = \lambda$$

and:

$$s_k = \exp\left[a\left(\pi_k - c_k\right)\right]\exp\left[-\left(a\lambda+1\right)\right].$$

Equation (A2) and the previous equation imply that:

$$\sum_{m=1}^{K}s_m = \left[\sum_{m=1}^{K}\exp\left[a\left(\pi_m - c_m\right)\right]\right]\exp\left[-\left(a\lambda+1\right)\right]=1,$$

$$\exp\left[-\left(a\lambda+1\right)\right]=\left[\sum_{m=1}^{K}\exp\left[a\left(\pi_m - c_m\right)\right]\right]^{-1},$$

and finally that:

$$s_k = \frac{\exp\left[a\left(\pi_k - c_k\right)\right]}{\sum_{m=1}^{K}\exp\left[a\left(\pi_m - c_m\right)\right]}.$$

Appendix 2: Acreage shares in the nested MNL model

In the nested MNL case, the producer's program is provided by:

$$\underset{\mathbf{s}}{Max}\sum_{q=1}^{Q}\sum_{m\in B(q)} s_m\left(\pi_m - c_m\right) - a^{-1}\left(\sum_{q=1}^{Q}\overline{s}_q \ln\overline{s}_q + \sum_{q=1}^{Q} a\alpha_q^{-1}\overline{s}_q \sum_{m\in B(q)} s_{m/q}\ln s_{m/q}\right) \text{ s.t. } \sum_{k=1}^{K} s_k = 1$$

with:

$$\overline{s}_q = \sum_{m\in B(q)} s_m \text{ and } s_{m/q} = s_m / \overline{s}_q \; .$$

The corresponding Lagrangian function is defined by:

$$L(\mathbf{s},\lambda) = \sum_{q=1}^{Q}\sum_{m\in B(q)} s_m\left(\pi_m - c_m\right) - a^{-1}\left(\sum_{q=1}^{Q}\overline{s}_q \ln\overline{s}_q + \sum_{q=1}^{Q} a\alpha_q^{-1}\overline{s}_q \sum_{m\in B(q)} s_{m/q}\ln s_{m/q}\right) - \lambda\left(\sum_{k=1}^{K} s_k - 1\right).$$

The FOCs for the crop k in nest q are provided by:

(A3) $$\frac{\partial L}{\partial s_k} = \pi_k - c_k - a^{-1}\left(\ln\overline{s}_q + 1\right) - \alpha_q^{-1}\ln s_{k/q} - \lambda = 0$$

(A4) $$\frac{\partial L}{\partial \lambda} = \sum_{k=1}^{K} s_k - 1 = 0$$

Equation (A3) leads to:

(A5) $$\alpha_q^{-1}\ln s_{k/q} + a^{-1}\ln\overline{s}_q = \pi_k - c_k - a^{-1} - \lambda$$

Equation (A5) allows to show that:

$$\alpha_q^{-1}\ln s_k - \alpha_q^{-1}\ln\overline{s}_q + a^{-1}\ln\overline{s}_q = \pi_k - c_k - a^{-1} - \lambda$$

and, as result, that;

$$s_k = \exp\left[\alpha_q\left(\pi_k - c_k\right)\right]\exp\left[-\alpha_q\left(a^{-1} + \lambda\right)\right]\exp\left[\left(1 - \alpha_q a^{-1}\right)\ln\overline{s}_q\right].$$

Using the definition of $\overline{s}_q$, *i.e.*, $\overline{s}_q = \sum_{m\in B(q)} s_m$ leads to:

$$\overline{s}_q = \exp\left[-\alpha_q\left(a^{-1} + \lambda\right)\right]\exp\left[\left(1 - \alpha_q a^{-1}\right)\ln\overline{s}_q\right]\sum_{m\in B(q)}\exp\left[\alpha_q\left(\pi_m - c_m\right)\right]$$

and, as result, to:

(A6) $$s_k = \frac{\exp\left[\alpha_q\left(\pi_k - c_k\right)\right]}{\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]}\overline{s}_q$$

and:

(A7) $$s_{k/q} = \frac{\exp\left[\alpha_q\left(\pi_k - c_k\right)\right]}{\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]}.$$

Equation (A3) allows to show that:

$$\overline{s}_q = \exp\left[a\left(\pi_k - c_k\right)\right]\exp\left[-1 - a\lambda\right]\exp\left[-a\alpha_q^{-1}\ln s_{k/q}\right].$$

With $\sum_{q=1}^{Q}\overline{s}_q = 1$ we obtain:

$$\exp\left[a\left(\pi_k - c_k\right)\right]\exp\left[-1 - a\lambda\right]\sum_{q=1}^{Q}\exp\left[-a\alpha_q^{-1}\ln s_{k/q}\right] = 1$$

and finally:

$$\overline{s}_q = \frac{\exp\left[-a\alpha_q^{-1}\ln s_{k/q}\right]}{\sum_{q=1}^{Q}\exp\left[-a\alpha_q^{-1}\ln s_{k/q}\right]}.$$

Integration of equation (A7) leads to:

$$\overline{s}_q = \frac{\left[\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]\right]^{a\alpha_q^{-1}}}{\sum_{q=1}^{Q}\left[\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]\right]^{a\alpha_q^{-1}}}$$

and thus to:

$$s_k = \frac{\exp\left[\alpha_q\left(\pi_k - c_k\right)\right]}{\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]}\frac{\left[\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]\right]^{a\alpha_q^{-1}}}{\sum_{q=1}^{Q}\left[\sum_{m\in B(q)} \exp\left[\alpha_q\left(\pi_m - c_m\right)\right]\right]^{a\alpha_q^{-1}}} = s_{k/q}\overline{s}_q$$

where crop *k* belongs to nest *q*.

Appendix 3: Acreage share price elasticities in the standard MNL model

It can easily be shown that:

$$\frac{\partial s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial p_k} = a\frac{\partial \pi_k}{\partial p_k} s_k(\boldsymbol{\pi};a,\mathbf{c})\left[1 - s_k(\boldsymbol{\pi};a,\mathbf{c})\right] > 0$$

$$\frac{\partial s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial p_\ell} = -a\frac{\partial \pi_\ell}{\partial p_\ell} s_k(\boldsymbol{\pi};a,\mathbf{c}) s_\ell(\boldsymbol{\pi};a,\mathbf{c}) < 0$$

$$\frac{\partial s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial \mathbf{w}} = a s_k(\boldsymbol{\pi};a,\mathbf{c}) \sum_{m=1,\neq k}^{K} \left(\frac{\partial \pi_k}{\partial \mathbf{w}} - \frac{\partial \pi_m}{\partial \mathbf{w}}\right) s_m(\boldsymbol{\pi};a,\mathbf{c})$$

and:

$$\frac{\partial \ln s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial \ln p_k} = p_k a\frac{\partial \pi_k}{\partial p_k}\left[1 - s_k(\boldsymbol{\pi};a,\mathbf{c})\right]$$

$$\frac{\partial \ln s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial \ln p_\ell} = -a p_\ell \frac{\partial \pi_\ell}{\partial p_\ell} s_\ell(\boldsymbol{\pi};a,\mathbf{c})$$

$$\frac{\partial \ln s_k(\boldsymbol{\pi};a,\mathbf{c})}{\partial \ln w_j} = a w_j \sum_{m=1,\neq k}^{K} \left(\frac{\partial \pi_k}{\partial w_j} - \frac{\partial \pi_m}{\partial w_j}\right) s_m(\boldsymbol{\pi};a,\mathbf{c}).$$

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